ELECTRICITY AND MAGNETISM

FOR BEGINNERS

SANDERSON

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ELECTRICITY & MAGNETISM

FOR BEGINNERS

Phys. Elect.

ELECTRICITY & MAGNETISM
FOR BEGINNERS

BY

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WITH ILLUSTRATIONS

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PREFACE

The following lessons in Electricity are intended to form a first course for boys who have already learnt the elements of mensuration, statics, dynamics, and heat. As the object of the book is to introduce the student to the principal laws of the subject, and to give him a working knowledge of the quantities involved, I have thought it well, at this stage, to introduce only direct experiments. It will be advisable to omit the last three sections of the chapter on magnetism until the second and third chapters have been studied. The apparatus required for the experiments is described in the text, or is made clear by the diagrams. This apparatus is of the simplest character, and can be provided at a very small cost for classes of as many as thirty to work the same experiment at the same time.

F. W. Sanderson.

Oundle School,
April 1897.

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CHAPTER I

MAGNETISM

SECTION I

1. CERTAIN natural bodies have been found to possess the property of attracting small pieces of soft iron. These bodies were originally found in Magnesia, and are called magnets, and they are said to be endowed with magnetism. It has been found that this property can be imparted to other bodies, and in a large degree to iron and steel. Steel magnets are now made of various forms — the best known being the bar magnet, a prism form, and the horse-shoe magnet. Both ends of a magnet are capable of attracting soft iron, but the following experiments will show that the two ends differ from each other in an important manner.

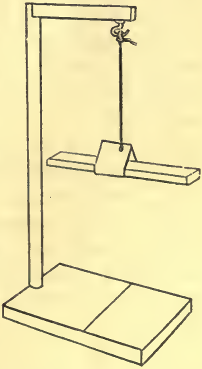


Fig. 1.

EXPT. 1.—Take a magnet and prove that both ends will attract soft iron tacks. Place the magnet in a paper stirrup and suspend it by a few fine silk fibres. It will be found that the magnet sets in a definite position, and that this position is approximately north and south. Turn the magnet until it is at right angles to the north and south, it will oscillate about its original position and finally come to rest in it. Take the magnet out of the stirrup, and replace it with its ends reversed, it will turn round and set with one particular end always pointing towards the north. This shows that the two ends of the magnet differ from each other. Mark with a piece of gummed paper that end of the magnet which points towards the north. The experiment also shows that there is a force pulling the marked end towards the north, and the unmarked end towards the south. It will be found that this force is due to the earth being a magnet; and if the magnet is not influenced by any other magnetic substance, it is said to be in the earth's field of force.

EXPT. 2.—Take a second magnet, place it in the stirrup, and mark the end which points towards the north. Now bring the marked end of the first magnet near to the marked end of the second magnet, which is left suspended in the stirrup. It will be found that the marked end of the suspended magnet is repelled. Now bring the unmarked end near to the marked end of the suspended magnet—they will attract each other. Thus the two ends of the magnet possess opposite properties.

The end of the magnet which points towards the north is called the north or positive end: the end of the magnet which points towards the south is called the south or negative end.

The experiment shows that—(i) Like ends (*i.e.* either both positive or both negative) are repelled from each other; (ii) unlike ends (*i.e.* one $+^{\text{ve}}$ the other $-^{\text{ve}}$) are attracted towards each other.

Substances which are attracted to *both* ends of a steel magnet are called **paramagnetic** substances, or more briefly "magnetic" substances.

The chief paramagnetic substances are iron, nickel, and cobalt. There are substances which are repelled from *both* ends of a steel magnet. These are called **diamagnetic**. The two more important diamagnetic substances are bismuth and antimony.

What is the cause of the attraction and repulsion is still a subject of inquiry. The two opposite properties may be the different aspects of the same cause. For example, if a spiral made of a piece of glass tubing be rotated in water—water will be drawn in at one end and emitted at the other—one end is a source, the other a sink. If such a spiral could be set in rapid rotation in a stream of water, it would turn so that the water would flow naturally into the entrance end. One end might be called the marked end, and the other the unmarked end. Again, the effect at a point of the stream might be said to be due to something at the ends, because the effect at a point can be expressed in terms of the position of the ends and the rate of flow in and out. If, therefore, the effect at a point is said to be due to the magnetism at the ends, this must not be taken as conveying anything more than that the effects are related to the ends.

It will be seen afterwards that all substances are magnetic, and that the attraction or repulsion is a purely relative question, but for the present we shall consider iron and steel only.

EXPT. 3.—Take a bar magnet and prove that although the ends attract soft iron tacks the middle does not do so. The student can fix the magnet as in the figure, and suspend rows of tacks from different points of it. It would appear at first sight that the bar had no magnetism at the centre. If, however, the bar were broken at the middle it would be found to possess the magnetic property, and tacks could be suspended from it. This may be verified in the following way:—Take a watch spring and magnetise it by drawing the north end of a magnet along it two or three times, taking care to go from end to end. Find its north end by observing its effect upon the north end of a suspended or pivoted magnet. Mark the north end. Join the two ends carefully together so as to form a ring, and prove that it now attracts both ends of the magnet, and therefore appears as a magnetic substance only. Now break it in the middle, and test the magnetism of the ends of each half. Each part will be found to be a magnet with north and south ends. In the same way, if each part be also broken into two, each of the four parts will be found to be a magnet with north and south ends.

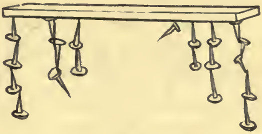


Fig. 2.

This experiment suggests that a magnet may be regarded as made up of a large number of very small magnets arranged in parallel chains, with their north and south ends placed together, and the next experiment will show how a large number of very small magnets can be arranged so as to behave as one simple magnet.

If we imagine the air and the medium to be itself magnetic, then the molecules of the medium will be arranged in definite order, and thus the magnetism will be propagated from point to point. If a piece of soft iron be placed in the field it will be magnetised, *i.e.* the magnetic molecules will be arranged in definite directions.

EXPT. 4.—Fill a test-tube with steel filings, and magnetise the filings by drawing the end of a magnet several times along the tube. The filings will not only have been magnetised, but will have had all their north ends turned towards one end of the tube, and the south ends set in the opposite direction. Now take the test-tube carefully, so as not to shake the filings, and test its ends; they will be found to be respectively north and south. Shake the filings, and again test the tube. It will now behave as a piece of unmagnetised steel.

When a piece of steel, say a knitting needle, is magnetised by drawing a north end of a magnet along it, the magnetic molecules have their south ends turned in the direction of motion of the magnetising end. Hence the end towards which the motion is made will be endowed with south magnetism, the other end with north magnetism. If the needle be shaken about it will lose its magnetism, especially if it is not very hard.

The difference between steel and soft iron can be explained by a modification of Expt. 4. If the filings were imbedded in a viscous substance like tar, it would be more difficult to arrange the magnets in the direction of length of the test-tube, but when once arranged it would be more difficult to disarrange them. In the case of steel it requires a strong magnetic force to arrange the magnetic molecules in the same direction; but when once they are arranged the steel maintains its magnetism. In the case of very soft iron the molecules are easily arranged, *i.e.* the iron is easily magnetised; but when the magnetising force is removed, the particles quickly return to their original position, and the iron quickly loses its magnetism. Anything which makes the particles move more easily will make magnetising more easy. For example, raising the temperature of the steel, and then allowing it to cool under the influence of magnetic force; or shaking the steel by hammering it whilst under the magnetising influence.

2. **Induction.**—If a short bar of iron be held near one end of a magnet, say the north end, the magnetic molecules of the iron will be arranged so as to have their south ends turned towards the north end of the magnet. The iron, whilst near the magnet, is itself a magnet. Now take the iron away from the magnet, and test it. It will have lost its magnetism. It is only a magnet when under the influence of another magnet, *i.e.* it is *induced* to be a magnet by a magnetic force. The iron is said to be magnetised by *induction.*

EXPT. 5. To show the Effects of Induction.

(*a*) Take a soft iron nail, say three inches long. Place the north end of a magnet about three inches from one end of a small pivoted magnet (needle), and notice the effect on it. Now bring the nail with its head near the north end of the magnet, and notice the effect of the other end of the nail on the suspended magnet. It will be found that north magnetism is induced in the end of the nail farthest from the north end. Now bring the north end of the magnet close up to the south end of the needle, and then carefully bring the head of the nail up to the needle, keeping the other end of the nail away from the magnet. The head of the nail will be found to slightly repel the south end of the needle, showing that south magnetism is induced in the part of the soft iron which is nearest to the north magnetism of the inducing magnet.

(*b*) Suspend a bundle of soft iron wires in a stirrup, and notice that it will set in almost any position. Place it in the north and south direction. Bring the north end of a magnet a little below the end of the wires, which points towards the north. Magnetism will be induced in the wires, and the south end will be pointing towards the north, and will be held there by the north pole of the inducing magnet. Gradually lower the north pole so as to remove it farther from the bundle; the bundle will begin to turn so as to set with its south end towards the north, thus proving that the bundle of wires is made a magnet by induction.

(*c*) Suspend two long soft iron wires vertically by means of threads, and bring a magnet up near to them from below, they will repel each other.

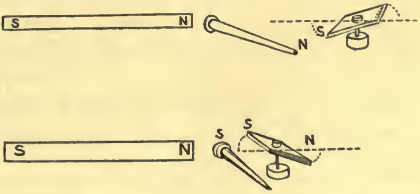


Fig. 3.

(*d*) Show, by placing sheets of wood, paper, or copper between the magnet and the soft iron, that the induction can take place through, and is not affected by, any non-magnetic substance. Show that this is not the case when a sheet of iron is placed between them.

Induction explains why a magnet always attracts soft iron.

The iron is made a magnet by induction, and by the nature of induction its south magnetism is always nearest to the north magnetism of the inducing magnet, so that they attract each other.

Steel can also be magnetised by induction, but not so easily as soft iron. Induction will be better understood after reading Section II.

Expt. 6. **To Magnetise a Piece of Steel.**

1st Method.—Single Touch.—Take one end, say the north, of a strong magnet, and move it along the steel from one end to the other, repeating this operation several times always in the same direction. Test, to prove that the end from which the motion takes place is the same kind as the north end, and that the other end is south.

2nd Method.—Separate Touch.—Take two magnets, and place the north end of one and the south end of the other together at the centre of the steel, and move them simultaneously in opposite directions to the ends of the steel. Repeat this operation several times. The steel will be more strongly magnetised if its ends rest on opposite poles of two magnets; or between the poles of a horseshoe magnet.

3rd Method.—By Electric Currents.—(See Chapter II).

Care must be taken to magnetise the steel uniformly. The effect of want of care in this respect can be shown by Expt. 7.

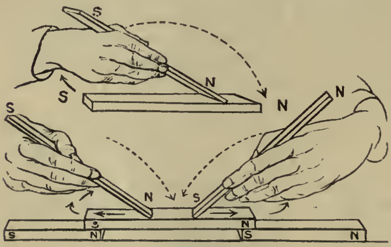


Fig. 4.

The steel for a magnet should be hardened. To do this make the steel red hot in a Bunsen flame, and cool it suddenly by dipping it into cold water.

Expt. 6a. **To Demagnetise a Magnet.**

Wind a piece of wire round the magnet, and make it red hot in a Bunsen flame, and then, holding it at right angles to the meridian near the flame, allow it to cool very slowly. It will be found to have lost its magnetism.

Expt. 7. **To prove the Existence of Consequent Points.**

Take a steel needle and magnetise it in four parts as above, but do not draw the magnetising magnet the whole length of the steel bar. Place it on a sheet of paper, and sprinkle fine iron filings over it. The filings will arrange themselves along the needle in such a manner as to demonstrate the existence of poles at points other than the ends. Make a freehand drawing of the iron filings, and mark the north and south poles. These points are called **consequent points.**

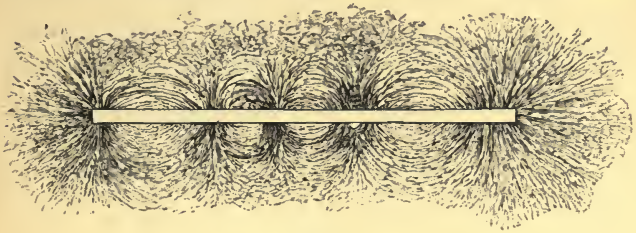
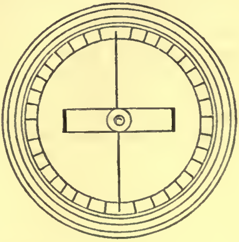


Fig. 5.

3. In the experiments which are given in the succeeding chapters, use will be made of an instrument which may be called a **Magnetometer.** It consists of a magnet 1·5 cm. long, pivoted on a glass centre, and provided with a brass indicator moving round a graduated circular disc. 10 cm. diameter placed in a mahogany case. By this means the deflection of the magnet can be read to about $\frac{1}{4}°$. The pivot (a fine needle), about which the magnet turns, may not be quite in the centre of the disc, so that the readings of each end of the pointer will not be the same. To avoid the error arising from this, take the readings of each end, and half the sum will be the true reading. A more exact form of magnetometer is described in the Appendix.



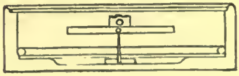


Fig. 6.

EXERCISES

I

1. A piece of iron wire is given to you. Explain how you would prove whether it were a magnet or not, and how you would find its north end.

2. Distinguish between a magnet and a magnetic body.

3. How could you discover whether a steel ring was magnetised or not?

4. Give as many methods as you can for testing whether a body is a magnet or not.

II

5. Explain the molecular theory of magnetism, and give an explanation of the magnetic difference between steel and soft iron.

6. How would you magnetise a knife-blade so that the point should be a negative end?

7. Show how to magnetise a needle so that its ends may be positive and centre negative—(i) by single touch, (ii) divided touch.

8. How would you magnetise a circular disc of steel of the size of a half-penny so that one face may be positive and the other side negative?

9. State the various conditions which facilitate the magnetisation of steel.

III

10. The north end of a magnet is held near the north end of another magnet, and is attracted by it. Explain this.

11. Explain why a magnet attracts soft iron.

12. How could a poker be made into a magnet with the positive end at the handle, the only magnetic field used being the earth's?

13. What would be the effect of placing a magnet with its south end pointing north, and hammering it?

14. An iron ship when building is placed north and south. What will be the general state of its magnetisation, and what distribution of magnets would counteract it?

15. A horse-shoe magnet is placed near a compass needle so as to pull the needle a little way round. On laying a piece of soft iron across the poles of the horse-shoe magnet the compass needle moves back towards its natural position. Explain this.

16. A glass tube, with its ends marked A and B, and nearly full of steel filings, is stroked several times from A to B with the north-seeking end of a strong magnet. The tube is then brought with its end B near to the south-seeking pole of a compass needle. What is the effect upon the needle? The tube is now shaken so as to mix the filings, and put near the needle as before. What is the effect upon the needle?

Why is the effect on the needle different in the two cases?

SECTION II

On the Poles and Axis of a Magnet—Lines of Force

1. **Poles and Axis of a long thin Magnet.**—Take a long thin magnet (NS) which has been carefully magnetised in the direction of its length, and place it on a horizontal sheet of paper. Now take a thin piece of soft iron wire, say 1 inch long, and suspend it by a fine fibre

attached to its middle point. With this investigate the direction of the force due to the magnet at points near the magnet. It will be found near the ends that the piece of wire sets always towards the end of the magnet, but that away from the ends no force appears to act on it. Now replace the piece of iron wire by a short steel magnet. It will be found that at the ends the little magnet sets with its length towards the end of the fixed magnet. At the centre it will be pulled towards the magnet, but this can be shown to be due to the bar magnet being magnetised by induction by the suspended magnet; for if we move the suspended magnet away from the fixed magnet, it will set with its length parallel or nearly parallel to NS. If the long magnet be suspended in a stirrup by a few silk fibres, it will be found that the ends appear to be attracted by soft iron, attracted or repelled by the end of another magnet. These experiments tend to show that a long thin magnet acts, and is acted upon, just as if its two ends were centres of force. *These centres are called the poles of the magnet, and the line* NS *joining them is called the axis of the magnet.* If the short suspended magnet be placed at a point P near the end N of the magnet NS and the pole of another magnet be brought up near to P, then we may assume—what future experiments confirm—that the force acting on the suspended magnet is the resultant of the force due to N and that due to the new pole.

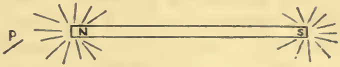


Fig. 7.

The axis of a long thin magnet always sets in a fixed direction at any place, under the influence of the earth alone, and this direction is roughly north and south, and is said to be in the magnetic meridian.

If several long thin magnets be joined together with similar poles at the same end and their axes parallel, so as to form one large magnet, then the magnet will set with the axes in the meridian, but the magnet will not act and be acted upon as if there were centres of force at its ends; in fact, it will not have poles. However, it is found that a bar magnet which is not too broad will act at a distance, and be acted upon by magnets at a distance—*i.e.* outside a certain surface surrounding the magnet

—as if there were centres of force near the ends, and these "centres of forces" are called the poles of the magnet.

If any magnet, whether it has poles or not, be suspended freely, there will always be found a line in it which sets in a given direction however the magnet be placed. The direction of this line is called the magnetic axis of the magnet. The plane which passes through the axis of this magnet, and through the vertical line, is called the **magnetic meridian** at the point.

EXPT. 1. To find approximately the "Poles" of a Bar Magnet.

Place the bar magnet on a sheet of paper, and take a small compass needle and mark the direction in which its axis sets—at two or more points near one end of the magnet. Produce these directions, and they will be found to meet approximately at a point within the magnet. This point may be taken as the pole of the magnet. In a similar way the pole at the other end can be found. The student will learn that this method may be briefly given as follows:—Draw the lines of force in the neighbourhood of the ends of the magnet, and find the points from which these lines appear to radiate.

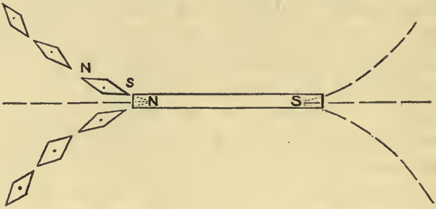


Fig. 8.

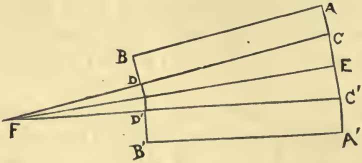


Fig. 9.

EXPT. 2. To find the Direction of the Axis of a Bar Magnet.

Suspend the magnet above a sheet of paper with its larger face horizontal in a stirrup by as few silk fibres as possible, and mark carefully on the paper the position of the magnet as in ABCD.

Now turn the magnet over, and again mark on the paper its position, of rest as A′B′C′D′.

If there is a line in the magnet which has always the same position in space, the position A′B′C′D′ can be obtained by turning the magnet about this direction. Produce AC and A′C′ to meet in E, and CD, C′D′ in F. Join EF, then if the magnet is turned about EF, AB will take the position A′B′, and CD the position C′D′. And any line in the magnet parallel to EF will remain parallel to EF. Hence EF is parallel to the axis of the magnet. Any line drawn parallel to EF through the magnet whilst in position ABCD, or A′B′C′D′, will be parallel to the axis of the magnet.

Place the magnet on EF, and with one side parallel to AB, and mark on the magnet (in pencil) a line along EF.

Mark the direction of EF on the paper. This is the line through the magnetic meridian.

The angle this direction makes with the geographic north and south is called the **angle of declination.**

To find the angle of declination it will be necessary to find the true north and south. This may be done roughly in the following way:—

Take an upright rod fixed to a base, and furnished with a fine wire at the top, or better with a disc through which a hole is bored. Place the stand on a sheet of paper, and at intervals, just before and just after noon, mark the position of the shadow of the top of the wire, or of the hole, and draw the curve connecting these points. It will be found convex to the upright. Now find the point on the paper vertically below the top of the wire, or the hole, and join this point to the vertex of the curve. This line will be in the geographic north and south.

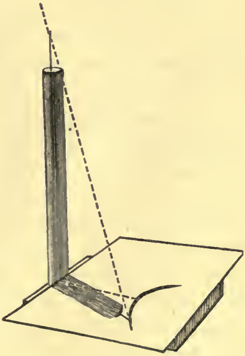


Fig. 10.

Note.—The ratio of the length of the shadow to the height of the needle is the tangent of the sun's zenith distance.

2. **Magnetic Dip.**—The following experiments will show that the earth exerts a vertical force on each end of a magnet, as well as a horizontal force :—

EXPT. 3. **To show that the Earth exerts a Vertical**

Force.—Take a piece of unmagnetised steel about 4 inches long (a piece of hardened knitting needle), and suspend it by a single silk fibre so that it rests horizontally. Now magnetise it carefully without disturbing the point of suspension. Suspend it by the fibre. It will be found to set in the magnetic meridian, and with its axis making a considerable angle (67°) with the horizontal, the north pole pointing downwards. The angle the axis makes with the horizontal is called the **angle of dip.**

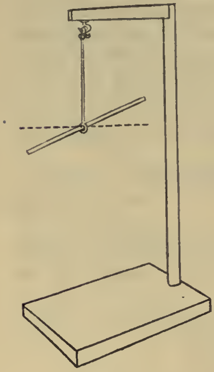


Fig. 11.

The angle of dip, like the angle of declination, varies from point to point on the earth's surface, and changes at every point with time.

An instrument consisting of a magnet carefully pivoted about a horizontal axis through its centre of gravity, and provided with a vertical graduated disc used for measuring the angle of dip, is called a Dip Circle.

3. **The Magnetic Action of the Earth on a Short Magnet** can be shown to consist of two equal, opposite, parallel forces, *i.e.* to form a couple.

i. Place a bar magnet on a cork floating in water, and wait until the water is quite still. The magnet will turn round and set with its axis in the magnetic meridian, but will not be pulled bodily in any definite direction. In fact it can be made to rest with the cork in any position on the surface of the water. If, on the other hand, a north end of a strong magnet be placed near the edge of the vessel, the floating magnet will be pulled so that its south end is brought as near as possible to the north end of the strong magnet. Hence we may take this as a proof that the earth's horizontal forces on the poles of the magnet are equal, parallel, and opposite.

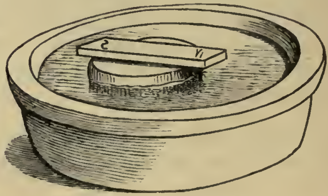


Fig. 12.

ii. Take an unmagnetised piece of steel and weigh it, then magnetise it and weigh it again. No difference will be found in these weights. Hence the earth's vertical force on the north pole is equal and opposite to its vertical force on the south pole.

Field of Force. Lines of Force.—If a short thin magnet be suspended by a fibre, and placed in the neighbourhood of a bar magnet (say), its two ends will be acted upon by forces tending to move them. If the suspended magnet be very short these two forces will be equal and parallel, and the magnet will set with its axis in the direction of the forces, because the magnet will turn until the two forces acting upon it coincide in position. A space which is such that at every point of it there is magnetic action is called a Magnetic Field of Force. If the action is due to a given magnet the field is called the Field of Force of the magnet.

A short magnet suspended by a long silk fibre furnishes a convenient method for determining the direction of the force acting upon a north pole placed at a given point of a field, and by its use a line can be drawn in the field such that the tangent at any point of it is the direction of the force on a north pole placed at the point. Such a line is called a *Line of Force.*

If, therefore, we could have a north pole free from the south pole, it would move along a line of force. This can be partly realised in the following manner:—Fix a long thin magnetised needle in a piece of cork, and float it in water with the north pole upwards and its axis vertical. It will rest at any point in the still water at which it may be placed, because the earth's force on the north pole is equal and opposite to that on the south. If, however, a *north* pole is brought up to the magnet just above the north pole, the force due to this *north* pole will be greater on the north pole than on the south, and the north pole will move along a line of force due to the magnet. In this way the form of the line of force due to north pole; two north poles; a north and south, can be seen.

Experiments on Lines of Force

To draw a Line of Force.—Take a short magnet suspended by a long fibre (but for a first experiment an ordinary pivoted compass needle will do) and hold it over a given point, and mark on the paper the position of its north and south poles at A and B say. Now bring the magnet so that the north pole is over B, and mark the position C at which the south pole rests. In this way the line of force can be drawn from point to point of the field.

Some idea of the lines of force due to a magnet can be obtained by means of iron filings. Sprinkle a large number of small iron filings uniformly on a sheet of paper or glass, and place the magnet in position. The iron filings will be made magnets by induction, and if the force acting upon them is great enough they will set with their lengths along the lines of force. The earth's magnetic force is not sufficient to magnetise the filings and set them in the direction of its lines of force, and therefore this is a convenient method of obtaining the lines of force due to a magnet *alone*. It will be seen from the following experiment, and a reason for it will be given afterwards, that if a piece of soft iron be placed in a magnetic field it will be magnetised in the direction of its greatest length—*i.e.* the axis of the magnet formed by *induction* is on the whole along the greatest length—hence the filings tend to set with their lengths along lines of force. It is important to notice, however, that the introduction of the iron filings changes the condition of the field, and that the lines of force due to a magnet need not be the same in air as they are when there are iron filings. This is easily seen by considering that soft iron will increase and change the direction of the force at a distance from the magnet (*cf.* Expt. 5, § 1). The line of force exhibited by the iron filings will only coincide with the line of force as would be drawn in air, if the filings are quite uniformly distributed over the whole field, and the magnets (created by induction) set actually along lines of force.

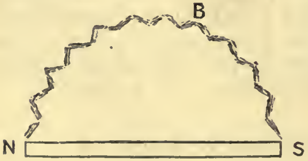


Fig. 13.

If we imagine the magnet surrounded by iron filings then a line could be drawn in the field which should pass along the magnet axes of the iron filings. For example, suppose NBS is a chain of these filings, then the zigzag line formed by the filings is a line containing the axes of the filings. Such a line need not coincide with a line of force, because the filings are held in position by the friction of the paper.

A line drawn in a field of force along the axes of the magnetic molecules is called a **Line of Induction.**

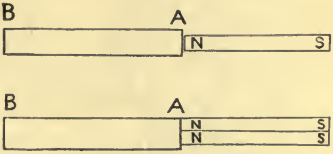


Fig. 14.

If a piece of soft iron be placed at AB, we may suppose it to consist of a very large number of small magnets arranged anyhow. Now bring up the magnet NS. The effect of this will be that some of the south poles near A in the soft iron will be turned towards N, and this will be propagated towards B, so that B will become magnetised positively. If now a second magnet be placed on NS with its north pole on N, more of the molecules near A will have their south poles turned towards N, and this motion will be propagated from A towards B, so that A and B will be more highly magnetised. The south poles of the molecules are displaced towards A, and the north poles towards B, and it is this displacement of the molecules that transmits the magnetic force along AB. The magnetic force is propagated by induction, which is caused by this displacement. All magnetic effects are produced by this displacement in the medium. The molecules of the air, or of the ether pervading the air, are magnetic, and when a magnet is placed in the air the molecules are turned round with their axes arranged in fixed directions, viz. along a line of induction.

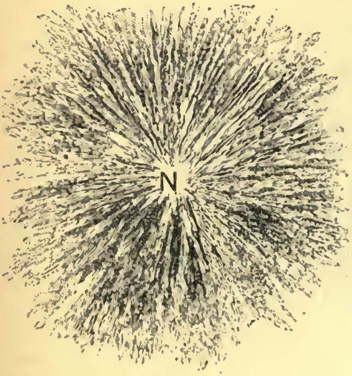


Fig. 15.

I. Lines of Force due to Magnets alone—neglecting the Earth's Magnetic Force.

These lines can be drawn by means of iron filings as explained above.

(*a*) *Draw the lines of force due to a single pole* (Fig. 15).

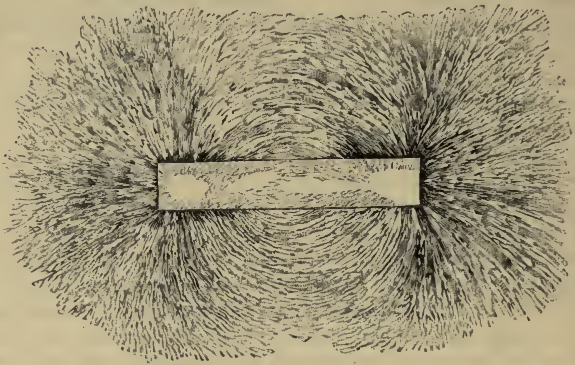


Fig. 16.

Fix a sheet of glass horizontally about nine inches above the table. Soak a sheet of paper in melting paraffin, and place the sheet on the

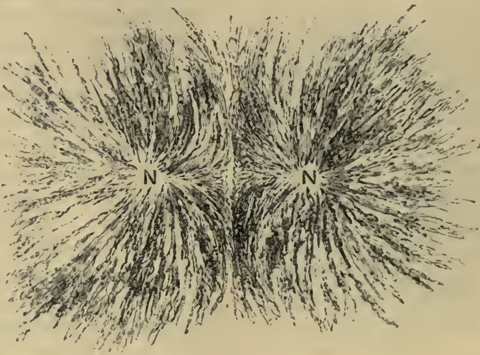


Fig. 17.

glass. Now fix a magnet vertical, with its north pole touching the sheet of glass about the centre of the paper. Strew fine iron filings uniformly over the paper, and gently melt the paraffin by means of a small flame from a Bunsen burner, or by steam from boiling water, applied below the glass. Tap the glass so as to enable the iron filings to move and set in the lines of force. Now allow the paraffin to cool, and thus fix the filings in their place.

These lines are found to be straight lines radiating from the pole.

(β) *Draw the lines of force due to a north and south pole* i.e. *due to a bar magnet* (Fig. 16).

(γ) *Draw the lines of force due to two north poles, placed say* 4″ *apart* (Fig. 17).

II. Lines due to the Earth, and to Magnets placed in the Earth's Field.

(α) *Draw the lines due to the earth's field only.*

These lines will be found to be parallel and straight. The field is therefore called a parallel field of force. If the times of vibration of a magnet be taken at different points of the field, they will be found equal. Hence we may conclude—and this will be abundantly verified by other methods—that the force exerted on the same north pole is the same at all points of a parallel field.

(β) *Draw the lines of force due to a bar magnet placed with its axis in the meridian, and having the south pole towards the north* (Fig. 18).

Find the magnetic meridian, and place the magnet with its axis in it, but at such a distance from the edge of the paper that the edge of the paper is practically outside the magnetic field of the magnet. Now divide the line at right angles to the meridian and near the edge of the paper into a large number of equal parts, say ·5 cm. apart, and begin drawing the lines of force from these points, so that when the lines are equidistant the forces are the same. It will be found that as the lines get nearer the intensity increases.

Near the south pole of the magnet the north pole of the compass magnet is attracted towards the south pole, *i.e.* towards the south, but this force becomes less as the compass is moved along SN. Hence somewhere in the axis produced the forces due to the magnet and to the earth respectively become equal, and since they are opposite they will balance each other. Hence somewhere in the axis produced there

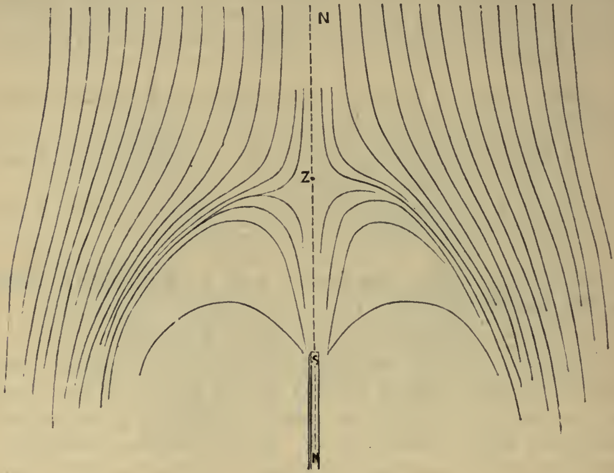


Fig. 18.

will be a **zero** or **neutral** point, *i.e.* a point where there is no magnetic force.

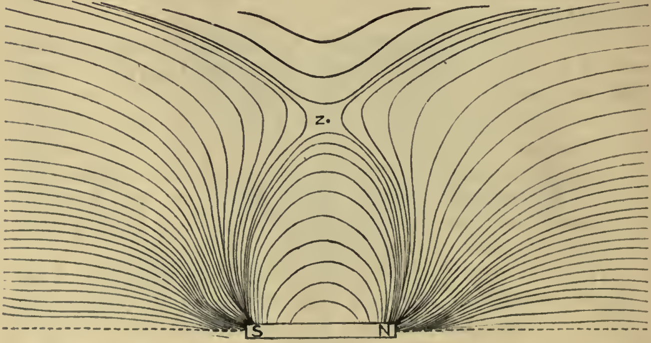


Fig. 19.

(γ) *Draw the lines of force due to a bar magnet placed* *with its axis in the magnetic meridian, having its north pole towards the north* (Fig. 19).

(δ) *Draw the lines of force due to a bar magnet placed with its axis at right angles to the earth's field of force.*

III. Lines of Forces due to the Earth and Magnets Modified by the Presence of Soft Iron.

(*a*) *Draw the lines of force due to the earth, modified by a rectangular bar of soft iron placed with its length in the magnetic meridian* (Fig. 20).

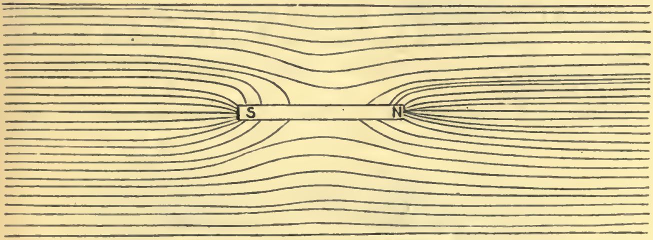


Fig. 20.

Note how the lines of force run down into the soft iron, and may be regarded as reappearing again.

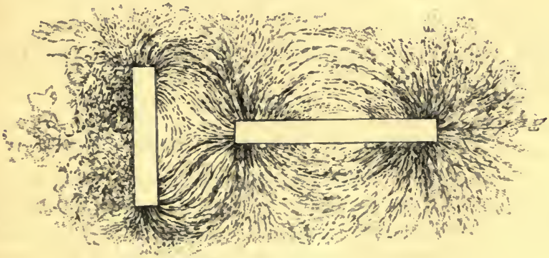


Fig. 21.

The parallel lines of the earth run down into the soft iron, and crowd together through it.

This is sometimes expressed by saying that "soft iron" **conducts lines of force** better than air, or to be more **permeable** to the lines of force than air.

Note also that the lines in the neighbourhood of soft iron radiate from it somewhat as they do from the magnet. The soft iron appears as a magnet, and is said to be a magnet by induction.

(β) *Draw the lines of force due to a bar magnet and a bar of soft iron placed in various positions.*

These figures show how the lines of force converge towards the soft iron and run through it.

Fig. 21 shows how the lines of force run through the soft iron, and thus there is no force on the other side of the iron. The soft iron acts as a magnetic shield, cutting off the influence of the magnet.

This is further shown in Fig. 22, where the lines of force are seen to leak from the two magnets, but the soft iron keepers act as shields.

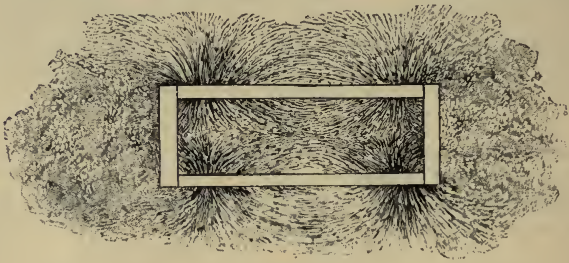


Fig. 22.

Properties of Tubes of Induction

I. The intensity decreases as the distance between the lines, or, more strictly, the area of the section of the tube bounded by lines of induction increases. Along a tube it will be shown that the intensity varies inversely as the cross-sectional area of the tube.

II. The tubes are in a state of tension. If as in Fig. 15 a north pole is removed from the magnetic field, its lines will radiate uniformly from the pole. If now a south pole be brought up, as in Fig. 23, the effect will be that the tubes are pulled round towards the south pole. There is therefore a greater number of tubes pulling N towards S than there is pulling it in the opposite direction. The two poles will therefore be pulled together, as experience tells us is the case. If a N pole were brought up to the first north pole, the tubes would be driven round towards the other side, and more tubes would pull the north pole on the side away from the second north pole, than on the side towards it. Hence the two north poles would be pulled farther apart, *i.e.* repulsion would take place.

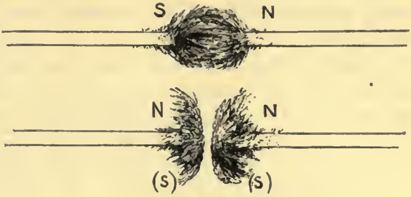


Fig. 23.

III. The tubes of induction repel each other laterally, *i.e.* transversely; hence the less the force at any point the farther the lines are from each other.

IV. The tubes of induction prefer (as it were) to pass through soft iron rather than through air, so that they bend down as much as possible and crowd into the soft iron, so as to pass along it as much as possible. Soft iron is said to conduct them better than air. Thus in Fig. 20 the lines bend down towards and pass through the soft iron. In Fig. 21 they do the same, and tend to pass through the length of the iron; in this case the pull along the lines will pull the soft iron towards the magnet. In Fig. 22 the lines pass through the soft iron, and none pass beyond it. Hence a thick iron cylindrical ring forms a good **magnetic screen.** The figure in Fig. 20 also shows how to the outside field the soft iron appears as a magnet. In this figure the cylindrical bar appears as a magnet with its poles near its ends. The tubes of induction and, therefore, the lines of force from a magnet may be directed by means of soft iron, and a conical pole-piece will collect them together and project them concentrated, as it were, at a given place, thus increasing the attractive force. *It can be shown that the attractive pulling force is proportional to the square of the density of the lines of force.*

V. The following general principles will also be verified on reference to the diagrams :—

(i) The magnet or soft iron tends to set so as to offer least resistance to the lines, *i.e.* so as to deviate the original field as little as possible; or, which is the same thing, so as to allow as many as possible of the original lines of the field to pass through it.

For example, in Fig. 18 the magnet will turn so that the lines of the earth may run into it and through it, and therefore be least distorted, *i.e.* the magnet will tend to set with its axis in the meridian, and its north pole towards the north, as in Fig. 19.

(ii) The magnet tends to move into the strongest part of the field. Thus in Fig. 17 the poles both tend to move away from the middle point between them, where the force is least. Similarly in Figs. 21, 24, 25. This is in agreement with the fact that the iron will move so as to embrace the greatest number of lines, or offer least resistance. The full meaning of this will be better understood when the student has read electro-magnetism.

VI. A permanent magnet sends out tubes of its own; and the student will find that a magnet may be regarded as having associated with it a definite number of tubes of induction radiating from it, and that the direction and not the number of these can be altered. Figs. 18, 19, 23 show this; in other cases, some of the tubes of the magnet run in the same direction as the earth's, and are apparently lost.

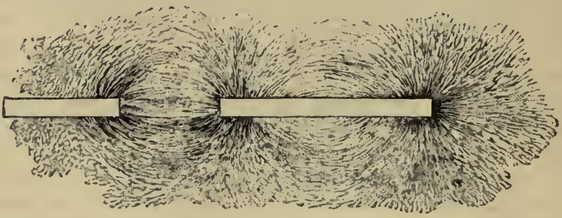


Fig. 24.

Magnetic Circuit.—Place a strong magnet NS on a sheet of paper and sprinkle iron filings around it. They will set along lines of induction. Now bring a bar of soft iron up to the south pole; it will be found that as it is brought nearer, the lines of filings bend round and crowd in upon the iron instead of going off to the north pole (Fig. 24); if the iron be turned at right angles, as in Fig. 25, the lines will bend down towards it, and filings will be found to spring from its other end. This is an illustration of the fact that the lines or tubes of induction seek the soft iron. The lines ultimately go through the soft iron on to the north pole of the magnet.

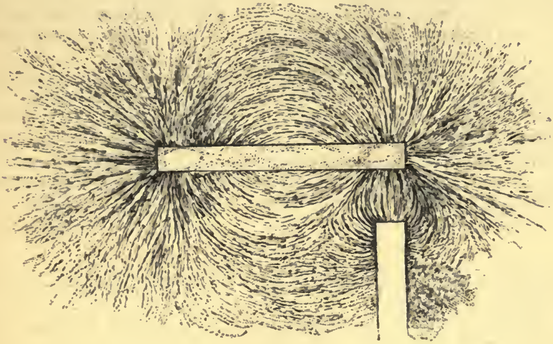


Fig. 25.

If a second piece of soft iron be placed on the north pole, very few filings will be found to set on the other side of the magnet; they are all concentrated at the ends of the soft iron. If now a bar of soft iron be connected across from one pole to another by means of cross pole-pieces of soft iron, the magnetic circuit will be now completed through this soft iron, and it will be found that the whole system exhibits very little magnetic force outside the magnet and soft iron.

This can be tested not only by means of iron filings, but also by suspending a magnet and noticing how freely it sets parallel to the axis of the magnet, then put on the connecting piece of soft iron and the magnet will be found to return almost to the magnetic meridian, showing that it is not acted upon by the bar magnet. The lines of induction now pass through the soft iron, and very few leak out, as it were, into the air space.

ILLUSTRATIONS

(*a*) If a soft iron ball be placed on the flat end of a magnet it will roll to the edge, so that the lines of force may be shortened in passing to the other pole, and pass through as long a length of iron as possible.

(*b*) Tacks suspended by their heads from a magnet will hang with only one edge touching the magnet, and not with the head flat on the magnet; in the first position the lines pass through the width of the head, in the second only through the thickness. The attractive force, being proportional to the square of the density of the lines, will also be greater because the lines are more concentrated.

(*c*) A long thin piece of soft iron will set with its direction along the lines of force and not across them, because in this position the lines may pass through as great a length of iron as possible. The piece of iron differs, however, from a magnet in the fact that it can be taken out of the stirrup and its ends reversed, when it will be found still in equilibrium. We may compare a long straight piece of wood floating down a stream. It will float lengthwise down the stream.

EXERCISES

1. Define "axis" of a magnet, and show how to find the direction in a bar magnet of its axis.

2. Define "magnetic meridian" and "geographical meridian," and show how to find the angle between them at any given place.

3. How would you determine the direction of the axis of a magnetised sphere?

4. How does a magnet-needle set in London if perfectly free to turn about its centre of gravity?

5. What experiments prove that the earth exerts equal and opposite forces on the poles of a magnet?

Two magnets, one large, the other small, are floated in separate basins of water. When a mass of iron is brought between these basins the large magnet floats towards the iron, while the small magnet remains at rest. Explain this.

6. What is a magnetic line of force? What information does a diagram of lines of force give at each point?

7. Distinguish between a line of force and a line of induction. What is a tube of induction?

8. What is meant by a uniform field of force?

9. Describe experiments illustrating the idea that tubes of induction are like stretched rods of india-rubber in a state of tension.

10. As the force becomes less along a tube of induction, the tube swells out, so that it exerts pressure on adjacent tubes, and appears to repel them. Give experimental illustrations of this.

11. A north pole repels a north pole. Draw the lines of force due to two north poles, and explain this repulsion on the assumption that tubes of induction are (1) in a state of pull, (2) that they repel or thrust against each other, (3) that a magnet tends to move into the part of the field where the force is greatest.

12. A cylindrical piece of soft iron is placed horizontally in the earth's field, its axis inclined to the meridian. Sketch the lines of force, and show that the forces acting on the soft iron tend to set it with its axis in the meridian.

13. Define magnetic circuit. Explain why in a hollow sphere of very soft iron there is no magnetic force.

14. What are keepers? Explain why keepers preserve magnets.

15. Two bar magnets are provided with keepers. Explain why the force necessary to pull off a keeper is greater when the other keeper is on than when it is off. Give drawings of the lines of force illustrating your answer.

16. Sketch the lines of force due to a bar magnet, marking with arrows the direction a north pole would move; and draw lines through (i) all points where a small compass-needle would set parallel to the axis, (ii) where it would set perpendicular to the axis.

17. A bar magnet is set parallel to the magnetic meridian (i) with its north pole towards the north, (ii) with its south pole towards the north. Draw a line through all points where the force is parallel and opposite to that of the earth's horizontal force. Hence find the number of points where the force of the magnet is exactly balanced by the earth's horizontal force.

18. A bar magnet is set at right angles to the magnetic meridian, Draw a line through all points where the magnetic force is parallel and opposite to that of the earth's horizontal force. Hence find the number of points where the force of the magnet is exactly balanced by the earth's horizontal force.

19. Draw a sketch of the lines of force due to a bar magnet placed in the magnetic meridian (i) when the south pole is towards the north, (ii) when the north pole is towards the north. How would you determine roughly in what parts to expect the zero points?

20. Draw a sketch of the lines of force due to a bar magnet placed at right angles to the magnetic meridian. How would you determine roughly in what parts to expect the zero point?

21. Two bar magnets are placed parallel to each other, with their like ends equally distant from each other. Draw a sketch of the lines of force.

22. How would you find the exact position of the axis of a bar magnet relative to the magnet?

23. A steel disc is magnetised with one face positive, the other negative. It is then suspended by the edge. How will it set? Draw approximately the lines of force due to it.

24. What is a Field of Force? Give an account of the earth's magnetic field of force. What is meant by the earth's horizontal magnetic force? Where is it zero, and where has it its greatest value?

25. Show how to construct a dip-needle. What causes the dip? Where is the dip of the needle greatest, and why?

26. Given a dipping-needle only, how could the magnetic meridian be found?

SECTION III

Magnetic Measurement

1. **On the Force between the Poles of Magnets. Torsion Balance.**—In order to investigate the law of force between two magnets, it is necessary to have some means of measuring the force which one magnet exerts on another, in terms of the units of force, *i.e.* some form of magnetic balance is required. Since a magnet when suspended in the earth's field of force so as to be capable of rotation about a vertical axis, is acted upon by a couple, the magnitude of the couple required to hold the magnet at a given angle to the meridian could be determined by attaching two fine cords to the ends of the magnet, and passing the cords over pulleys, and attaching equal weights to the ends.

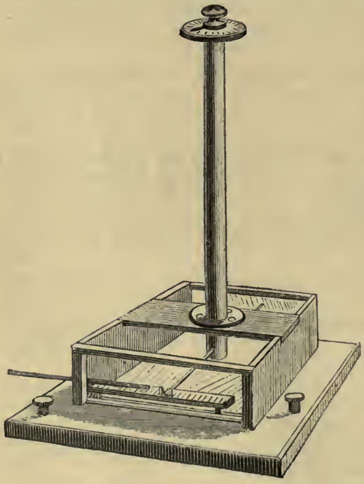


Fig. 26.

But since a magnet 20 cm. long magnetised strongly will require a couple to maintain it at right angles in the earth's field of force, the order of magnitude of which will be only 100 cm.-dynes, *i.e.* require only forces of five dynes at the ends, this statical method is not practicable except in case of a magnetic field which exerts a very great force.

The couple required, when the field of force is that due to the earth only, is very small; hence the balancing couple can best be applied by means of the torsion of a wire [a very fine silver wire is suitable]. It can be proved experimentally that the couple exerted by a twisted wire is proportional to the angle through which the wire is twisted; and the couple required to twist a wire of given length and section through a given angle, say the angle whose circular measure is 1, *i.e.* (about 57°), can be found by experiments either statical or dynamical, and this couple may be called the coefficient of torsion and may be denoted by τ. Statically all that is required is to suspend a horizontal pulley from the wire, and find the couple required to twist the wire say two or three times round; or the coefficients of torsion can be determined by oscillation methods which will be described in the Appendix. A table can therefore be made giving the couples exerted by the wire for every angle of twist.

An instrument based on these principles, *i.e.* consisting essentially of a magnet suspended by a fine wire, which can be twisted through a known angle, is called a **Torsion Balance**.

The following form of balance is convenient for the purpose. ABCD is a wooden box with a glass side in it. The top of the box supports a brass tube, the upper end of which is fitted with a graduated disc. A brass cylinder (of known mass and dimension) is suspended by a fine wire attached to a pin which passes through the centre of the graduated disc and can turn about its axis, being provided with a pointer to measure the angle the top of the wire is turned round. A magnet of hard steel about 20 cm. long and 1 mm. diameter is inserted into a hole perpendicular to the axis of the brass cylinder, with its centre at the centre of the cylinder so arranged that the whole hangs horizontally. On each side of the magnet are placed two stops, so that the magnet cannot move far from a given position. There is a hole in one side of the box through which magnets can be inserted with their axes at right angles to the axis of the suspended magnet, and there is a horizontal scale, so that the distance of the end of the inserted magnet from the north pole of the suspended magnet can be read if necessary. The instrument must be set so that the suspended magnet is in the magnetic meridian with its north pole towards the north, for in this position the earth exerts no couple on it.

To measure the force exerted by the pole of a magnet upon the north end of the suspended magnet at a given distance apart.

Place the given magnet in position with its north pole at the given distance (say 2 cm.) from the north pole of the suspended magnet, and twist the wire until the magnet is at rest between the stops, the couple thus required is known from the table made for the instrument. If P dynes is the force acting on the north end—and P will be perpendicular to the axis of the magnet—and $2l$ cm. the length of the suspended magnet, the moment of P is $P \times l$ cm.-dynes; this is found by the angle of twist θ to be $\tau\theta$ $\therefore$ $Pl = \tau\theta$, or $\boxed{P = \frac{\tau\theta}{l},}$ and the force is known in dynes.

Now τ and l are always the same for the same balance, and hence the force P is known in terms of θ.

The quantity $\frac{\tau}{l}$ may be called the **constant**, or **reduction factor** of the balance (see Ex. 1 below).

The instrument can thus be graduated once for all to give P for every angle of twist.

2. **On the Strength of Magnetic Poles.**—If a long thin magnet be magnetised in such a way that the lines of force radiate from its ends, and a point be taken near one end, the force at this point may be said to be due to a quantity of magnetism at the pole; and the torsion balance gives a means of testing the equality of magnetic poles.

(i) **Two poles may be defined to be equal if at the same distance they exert the same force on the north pole of the suspended magnet in the balance.**

Hence two poles can be adjusted to equality.

(ii) **We may say that one pole is double the strength of another if it exerts on the north pole of the suspended magnet at the same distance twice the force that the other pole exerts.**

(iii) **It may be proved by the torsion balance that the force on a given north pole, due to a number of poles, is the resultant of the forces due to each pole separately.**

These experiments form the basis of a dynamical method of measuring magnetism, for they furnish a criterion of equality of magnetisation, and also a method of defining quantity of magnetism.

All that is required now is a suitable unit of magnetism.

The C.G.S. unit is that quantity of magnetism which, placed at a distance of 1 cm. from an equal quantity, will exert on it a force of 1 dyne.[1] A pole which exerts on the unit pole a force of *m* dynes will have *m* units of magnetism. Conversely 1 unit will exert on *m* units at a distance of 1 cm. a force of *m* dynes, and therefore *m'* units will exert on *m* units at the same distance a force of *mm'* dynes. Hence the force between two quantities of magnetism is proportional to the product of the measures of those quantities.

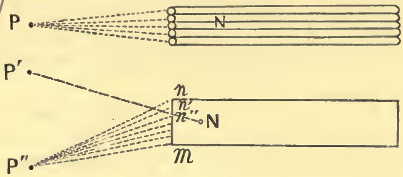


Fig. 27.

Any magnet may be supposed made up of a large number of magnets, and if the force due to each pole at a given point is known, then Expt. iii. enables us to apply the theorems of statics to determine the force due to all the magnets. This method is very valuable in some cases. For example, a long bar magnet may be built up from a large number of long thin magnets *sn*, *s'n'*, etc. The force at point P near the north end of the magnet is the resultant of the forces due to the poles *n*, *n'*, *n''* acting along *n*P, *n'*P, *n''*P, and this resultant will pass along a line PN.

If the point is at P' instead of P, the resultant will act along P'N'. If the point is at P'' the resultant will pass along P''N''. These three directions do not necessarily meet in a point. If P is at a considerable distance from the north end, so that all the lines P*n*, P'*n'*, etc., may be considered parallel and equal, then the resultant will always pass through the middle of *nm*, and the force at P is just as if all the magnetism were concentrated at this middle point. This point is the pole of the magnet. On account of the demagnetisation which takes place at the ends of a magnet, this point is really at a distance from the end. By the third law of motion the magnet, if placed in a field of force, will be acted upon by a force the direction of which passes through a point in it near N, and by a force the direction of which passes through a point S near the other end. These two points are called the poles of the magnet.

Ex. 1.—The length of the magnet in the torsion balance is 10 cm., and it requires a couple of 100 cm.-dynes to twist the wire through 360°: find the constant or reduction factor of the instrument. If a magnetic force would twist the wire through 60°, find the force.

---

[1] The torsion balance will test the equality of the poles, and will measure the force between them at a distance of 1 cm.

If P dynes is the force acting upon the magnet when the wire is twisted 360°, the moment of the force is P × 10 cm.-dynes, and this is equal to the couple due to the twist of the wire, *i.e.* to 100 cm.-dynes.

$$\therefore \quad P \times 10 = 100, \text{ or } P = 10 \text{ dynes.}$$

If the force required to twist the wire through $n°$ is P′ dynes, then

$$P' \times 10 = \frac{100 \times n}{360},$$

or

$$P' = \frac{1}{36}n,$$

$$\therefore \quad \frac{1}{36} \text{ may be taken as the reduction fraction in dynes.}$$

If $n = 60°$

$$P' = \frac{60}{36} = \frac{5}{3} \text{ dynes.}$$

$$= 1{\cdot}666 \text{ dynes.}$$

If the pole of the suspended magnet has a strength of 100 units then the intensity of the force = ·0166 dynes per unit.

Ex. 2.—It is found that when a north pole is brought to a distance of 1 cm. from the north pole of the torsion magnet, that the wire must be twisted 480°. If the two poles are of equal strength and the constant is $\frac{1}{36}$, find the strength of the poles in absolute units.

$$\text{Force between them} = \frac{1}{36} \times 480 = \frac{40}{3} \text{ dynes.}$$

$\therefore$ If m is strength of each pole,
force between them $= m^2$ dynes.

$$\therefore \quad m^2 = \frac{40}{3}$$

$$m = \sqrt{\frac{40}{3}} = 3{\cdot}65.$$

3. **On the Forms of Permanent Magnets.**—It will be convenient to describe here three fundamental forms of permanent magnets :—

(1) A long thin magnet, which is such that, it acts as if there were centres of force at its end. Such a magnet is called a solenoidal magnet, and the lines of force radiate from one end and converge into the other.

(2) A thin disc or shell magnet the disc being magnetised perpendicularly to its plane. So that we may say in the older language of magnetism, that one face is endowed with positive magnetism, and the other with negative. The lines of force due to such a magnet are something like those in the figure.

Here it is clear there are no "geometric" poles. This magnet has no poles, and therefore no length of axis.

The axis is perpendicular to its plane, and the magnetic moment would have to be found by finding the couple required to hold it in a field of unit intensity and at right angles to it (see Sections IV. and VII.)

(3) A bar magnet, as in Fig. 16, p. 16, where those lines of force which extend to some distance beyond the magnet radiate from points within the magnet. Such a magnet may, for points outside a certain surface, be replaced by an equivalent solenoidal magnet having its poles at these geometrical foci.

Consider the lines of force in Fig. 16.

It is easy to see what lines run approximately into two geometrical foci near the ends of the magnet. Hence at all points reached by these lines the magnetic force may be supposed due to two centres of force near the ends. These two centres of force may be called the "magnetic poles," and the same effects at a distance could be produced by a long thin magnet having those two points as poles.

It will be convenient to give here a brief definition of "magnetic pole" so far as we can.

It is found by experiments that a long thin uniformly magnetised bar of steel acts and is acted upon just as if it had centres of force at its ends. These centres are called the magnetic poles of the magnet, and it is mathematically convenient to regard these centres as endowed with equal but opposite quantities of "magnetism," capable of attracting or repelling quantities of magnetism at a distance.

A magnet which has practically definite centres of force is called a **solenoidal magnet.**

4. Experiments with the Torsion Balance.

(1) *Prove that the north and south poles of a magnet are of equal strength.*

(2) *Compare the strength of two poles.*—Place the two poles at the same distance from the torsion balance pole, and measure the angles θ, θ' of equilibrium, then,

$$\frac{m}{m'}=\frac{\theta}{\theta'}.$$

(3) *Find the strength in absolute units of a given pole.*

Magnetise a long rod of steel so that its poles are of equal strength with the given pole.

Now suspend this magnet in the torsion balance, and find the force between the given pole, and a pole of this magnet when 1 cm. apart. Suppose this force as measured by the balance is P dynes. Let the strength of each pole be m units, then the force between these poles is m^2 dynes.

$$\text{and}\qquad \boxed{\begin{gathered} \therefore\quad m^2=\mathrm{P} \\ m=\sqrt{\mathrm{P}}. \\ \\ \mathrm{P}=\frac{\tau}{l}\,\theta. \end{gathered}}$$

(4) *To find the couple which the earth exerts on given long bar magnet when its axis is at right angles to the meridian.*

Place the torsion balance so that the magnet is at right angles to the earth's field, and find the torsion required to hold it in equilibrium between the stops. This is the couple exerted by the earth's field on the magnet.

EXERCISES

1. What is a balance? Against what force is the magnetic force balanced in the torsion balance? How is this force measured? What measurement of the balance must be taken, and what previous experiment must have been performed to find this force.

2. What is the constant or reduction factor of a balance? Is a constant required in an ordinary spring balance? How is a spring balance graduated? Show how to graduate a torsion balance in the same way.

3. Show how to graduate a given wire with a magnet of given length as a torsion balance by direct statical methods.

4. Show how to find the coefficient of torsion of a given wire, *i.e.* the couple required to twist the wire through an angle whose circular measure is 1; and if the length of the magnet is 5 cm., find the forces corresponding to twists of 30°, 60°, 90°, 150°, the coefficient of torsion being 20 cm.-dyne. Find the constant of the instrument.

5. Show how to use the torsion balance to find the force at a given point of a field.

6. Why is it necessary to know the strength of the pole of the magnet in the torsion balance in order to find that of another pole? State fully (1) how you would proceed to create a magnet of unit strength of pole. (2) How you would determine the strength of the pole of the magnet in

the torsion balance. (3) How you would compare the strengths of two poles.

7. When are two magnetic poles said to be of equal strength? How would you prove experimentally (using a torsion balance) that the pole of one magnet was three times the strength of that of a given magnet?

8. What is the centimetre-gramme-second (C.G.S.) unit of magnetism? A long magnet is broken, one of these parts is suspended in the torsion balance, and a pole of the other is brought to within 1 cm. of pole of this part, and it is found that the force between them is 400 dynes. Show that the strength of each pole is 20 units.

Section IV

On the Equilibrium of a Magnet placed in a parallel Magnetic Field

1. **Intensity at a Point of a Magnetic Field.**—If a short but very thin magnet be placed in a parallel field of force, it will be acted upon by a couple, and since it is long compared with the thickness, it may be supposed to be acted upon by two equal and parallel forces at its poles. If the strength of each pole is unity then the force acting upon each pole will depend upon the strength of the magnetic field. The magnetic force acting upon the unit pole is called the "magnetic force at the point where the pole is" or "the **intensity of the field** *at* the point." It is expressed in dynes per unit quantity of magnetism. If the force on a unit quantity of magnetism at a point is H dynes—*i.e.* if the intensity at the point is H dynes per unit—then the force on m units at the point is Hm dynes.

The torsion balance gives a direct method of finding the intensity of the field *at* a point. Take a long thin magnet, the strengths of the poles of which have been found, and suspending it in the torsion balance, find by the torsion the couple required to hold it at right angles to the field. If the strength of each pole is m units, and the length of the axis $2l$ cm., the intensity of the field H dynes per unit, then the force on each pole is Hm dynes, and since the forces are perpendicular to the axis, the moment of the couple acting on the magnet is $\boxed{Hm \cdot 2l}$ cm.-dynes. Hence since m and l are known this will give H. Now the couple required to hold a magnet at right angles to a field will depend upon the magnet itself and upon the strength of the field. If this couple be known then the couple required to hold the magnet at an angle δ to the field can be found. (See Prop. 1, p. 35.)

2. **Magnetic Moment of a Magnet.**—The couple required to hold a magnet with its axis at right angles to a field of unit intensity is called the **magnetic moment of the magnet.**

If M is the magnetic moment of a magnet then the couple required to hold it at right angles to a field of intensity H dynes per unit is HM cm.-dynes. This couple can be found by the torsion balance, and dividing it by the measure of the intensity H, we find the measure of the magnetic moment.

If now the magnet is such that it may be regarded as having poles of strength m and axis $2l$, we have seen that the couple required to hold the magnet at right angles to a field of intensity H dynes per unit is $Hm \cdot 2l$ cm.-dynes. Therefore the couple required to hold a magnet of axis $2l$ cm. and strength of poles m units in a field of *unit* intensity is $2ml$.

Hence in the case of a magnet having poles the magnetic moment is proportional to the strength of the poles and the length of the axis.

It will be seen later in Sect. VI. that the intensity of the field at a given point due to a solenoidal magnet (m, $2l$) will depend upon the strength of the pole and the length of the axis, and if this intensity can be measured the magnetic moment may be found.

3. **Equilibrium of a Magnet placed in a Parallel Field.**—A bar magnet placed in a *parallel* field and free to move, will turn round until its axis is in the direction of the field, but it will not be translated (Cp. p. 12, § 3, i.). Hence the forces on its poles must be equal and opposite. Hence a magnet placed in a parallel magnet field is acted upon by a couple. The unit couple is called the cm.-dyne. To realise this unit couple take a lever 1 cm. long, capable of turning about one end, and apply a force of 1 dyne perpendicular to it at the other end. Now since a dyne $= \frac{1}{981}$ grm.-wt., *i.e.* roughly the weight of one millegramme, this couple is very small.

The English unit of couple is the inch-lb. wt.

i.e. a force of 1 lb. wt. acting at end of a lever 1 inch long,
or 1 oz. wt. ,, ,, ,, 16 inches long.
or $453 \cdot 59 \times 2 \cdot 5$ grm. wt. at end of lever 1 cm. long.
i.e.

$453 \cdot 59 \times 2 \cdot 5 \times 981$ dynes at the end of a lever 1 cm. long.

$$\therefore \quad 1 \text{ inch-lb. wt.} = 453 \cdot 59 \times 2 \cdot 5 \times 981 \text{ cm.-dynes.}$$
$$= 1,100,100 \text{ cm.-dynes (nearly).}$$

Hence 1 million (10^6) cm.-dynes would be a more practical unit of couple when dealing with the forces which occur in magnetic machines.

Take for example a motor turning a pulley 6″ diameter at rate of 1000 revolutions per minute, and transmitting $2\frac{1}{2}$ horse-power. The difference of forces on the parts of the belt round the pulley can be proved to be $4\frac{3}{8}$ lbs. wt., and the moment would be

$4\frac{3}{8} \times 3$ inch-lbs. wt.

i.e. $13\frac{1}{8}$ inch-lbs. wt.

The practical unit of couple taken is 10^7 cm.-dynes, and this is about

$\cdot 737 \times 12$ inch-lbs. wt.

i.e. about 9 inch-lbs. wt.

PROP. 1.—*To find the couple required to maintain a solenoidal magnet at a given angle in a field.*

Let NS be the positions of the two poles of the magnet, NS being the axis which makes an angle δ with the direction of the force of the field, *i.e.* parallel to *ns*. Let H be the intensity of the field, and *m* the strength of each pole — then force on each pole = H*m* dynes acting at N and S parallel to *ns*. Let NS = $2l$, O being the centre of the magnet about which it rotates. Let OL, OM be drawn perpendicular to the direction of the forces. Then the moment about O of H*m* acting at N is $Hm \times OM = Hm . l . \sin \delta$.

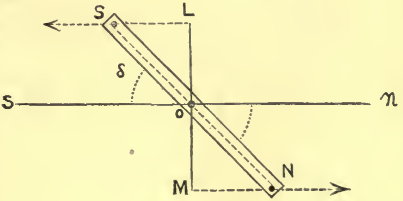


Fig. 28.

Moment about O of H*m* at $S = Hm \times OL = Hm . l . \sin \delta$.
Therefore the moment of the two forces $= 2Hm . l . \sin \delta$.

Hence if a magnet whose axis is $2l$ cm. long, and the strength of its poles m units, be placed in a field of intensity H dynes per unit in a direction making angle δ with the field, then the couple the field exerts upon it $= \mathrm{H}2ml \,.\, \sin\delta$.

The quantity $2ml$ is a constant for the same magnet; it is the **magnetic moment** of the magnet, for we see that it is measured by the couple required to maintain the magnet at right angles to a field of unit intensity, and therefore the magnetic moment is proportional to the strength of the poles and length of the axis.

If the magnetic moment is denoted by M, then the couple $= \mathrm{HM} \sin\delta$ and $\boxed{\mathrm{M} = 2ml.}$

PROP. 2.—*To find the condition of equilibrium when a short bar magnet is placed in a field due to two fields at right angles to each other.*

Suppose SN is the direction of one of the fields, and its intensity H dynes per unit, and EW be the direction of the other field, the intensity being F dynes per unit. To make the problem clearer, the first field may be supposed the earth's field, and the second may be supposed due to a bar magnet placed in such a position that its force at O is perpendicular to NS. Suppose ns is the position of the axis of the magnet when at rest under the forces due to the two fields. Let the angle it makes with NS *i.e.* $\mathrm{SO}s = \delta$; let $ns = 2l$, and strength of poles $= m$. The forces acting on the magnet due to H are $\mathrm{H}m$ at n and s in parallel but opposite directions; these give a couple $\mathrm{HM} \sin\delta$. The couple due to $\mathrm{F} = \mathrm{FM} \sin \mathrm{EO}s = \mathrm{FM} \sin (90° - \delta) = \mathrm{FM} \cos\delta$. If there is equilibrium these two couples must be equal.

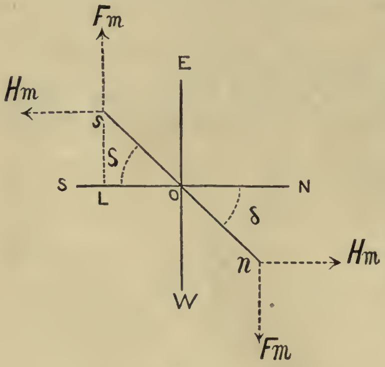


Fig. 29.

$$\therefore \quad \mathrm{FM} \cos\delta = \mathrm{HM} \sin\delta,$$

or

$$\mathrm{F} = \mathrm{H} \tan\delta.$$

This proposition is of so great importance that a direct proof will be given, and an experimental illustration.

Referring to the figure in Prop. 2, the forces acting at *s* are F*m* and H*m*, and since *s* is at rest, the resultant of these two forces must pass along O*s*.

Draw *s*L perpendicular to OS.

Then the triangle O*s*L has its sides parallel to the forces F*m*, H*m*, and the force along *os* acting at *s*.

Hence by triangle of forces

$$\frac{Fm}{Hm} = \frac{sL}{OL} = \tan \delta,$$

$$\therefore \quad F = H \tan \delta.$$

Experimental Illustration

NS is a rod 6″ long, pivoted between two centres at O, so as to turn about a horizontal axis. From the point S two strings pass over pulleys, and have scale pans attached at their other ends. The pulleys are so arranged that one string is horizontal and the other vertical; for this purpose one pulley can move up a vertical guide, the other along a horizontal guide as shown in the figure. A similar pair of strings are attached at

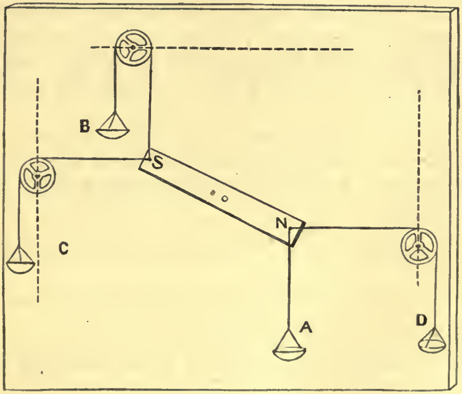


Fig. 30.

the end N, the forces along the strings acting in the opposite directions.

If W be the weight placed in the scale pans C and D, and W′ the weight placed in the scale pans A and B, and w be the weight of each scale pan.

Then each vertical force $= \mathrm{W} + w$,
and each horizontal force $= \mathrm{W}' + w$.

It will be found that

$$\frac{\mathrm{W}' + w}{\mathrm{W} + w} = \tan \delta,$$

where δ is the angle the beam makes with the horizontal.

Note.—If R dynes per unit is the resultant intensity at a point, then

(1) $\mathrm{R}^2 = \mathrm{H}^2 + \mathrm{F}^2$
(2) $\mathrm{R} = \mathrm{H} \sec \delta$
(3) $\mathrm{R} = \mathrm{F} \operatorname{cosec} \delta$

Ex. 1.—If the beam NS has a magnet of moment 5000 units fixed to it, and the whole be placed in a vertical magnetic field of intensity 8000 dynes per unit, what weight in grammes must be placed in the scale pans C, D so as to maintain the axis of the magnet at 30° to the vertical, the length of NS being 15 cm.?

The intensity of the field is 8000 dynes per unit, and the axis of the magnet is inclined to the direction of the field at an angle of 30°.

∴ The moment of the couple the field exerts on the magnet is

$$8000 \times 5000 \times \sin 30^\circ$$

i.e.

$$20,000,000 \text{ cm.-dynes.}$$

Hence it will be balanced by a force of $\frac{20,000,000}{15}$ dynes, having an arm of 15 cm. (*i.e.* 6 inches).

Hence force along the strings must be $\frac{4}{3}$ million dynes.

Now

$$981 \text{ dynes} = 1 \text{ grm.-wt.}$$

∴ Weight in scale pans (neglecting weight of the scale pans)

$$= 1359 \text{ grm.-wt.}$$

EX. 2.—Two magnets of magnet moments M and M′ are fixed with their axes at an angle to each other, but the axes being either in the same plane or in parallel planes, and the system is suspended by a fibre, so that the magnets can move freely in a parallel field of force—to find the position of equilibrium of the magnets :—

Let the axes of the magnets make angles respectively δ and δ' with the direction of the field—the intensity of which is H.

The couple acting upon the first magnet is HM sin δ, and on the second magnet HM′ sin δ'.

$$\therefore \ \mathrm{HM} \sin \delta = \mathrm{HM}' \sin \delta'.$$

$$\therefore \ \mathrm{M} \sin \delta = \mathrm{M}' \sin \delta'.$$

If the magnetic moments are equal

$$\sin \delta = \sin \delta',$$

$$\therefore \quad \delta = \delta',$$

or

$$\delta = 180^\circ - \delta'.$$

Hence the magnets will set so that one of the lines bisecting the angle between the axes is along a line of force.

This problem may be solved in the following manner :—

Let m, m' be the strengths of the poles of the magnets, then the north poles of the system are acted upon by the parallel forces Hm and Hm'; the south poles will be acted upon by the forces Hm and Hm' in the opposite direction.

Now Hm, Hm' are parallel forces, and their resultant will be H $(m + m')$, and will act at a point in the line joining the two north poles, and dividing it in the ratio of $m : m'$, *i.e.* at a point which would be the centre of gravity of particles of masses m, and m' placed at the poles.

In the same way the forces on the south or negative poles will have as resultant a force H $(m + m')$ parallel to H, and acting at the centre of gravity of m and m' placed at the negative poles.

Hence the line joining what may by analogy be called the centres of gravity of the north and south poles, must lie in the meridian.

This line is the axis of the system of the two magnets.

4. Experimental applications of relation $F = H \tan \delta$.

EXPT. 1.—To find the Magnetic Intensity at Points along the Axis of a Bar Magnet due to the Bar Magnet.

Draw a line in the magnetic meridian, and place the axis of the magnet perpendicular to the magnetic meridian. Draw a line on the paper along the axis produced. Mark off points at intervals of 5 cm. Place the magnetometer with its centre at one of these points. This can be done best by drawing two parallel lines equidistant from the axial line, their distance apart being equal to the diameter of the compass-box. The compass-box must then be placed between these lines, and the diameter of the compass-box being known, its centre can be placed at any given point. Now take readings for the deflection.

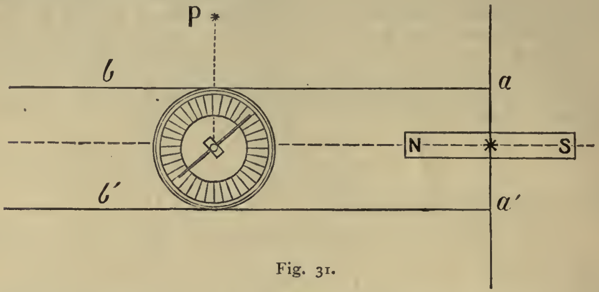


Fig. 31.

Ex. 1.—Readings $10\frac{1}{2}°$, $10°$. Therefore mean deflection $10\frac{1}{4}°$; now tan $10\frac{1}{4}° = \cdot 1854$. $\therefore$ $F = \cdot 18 \times \cdot 1854$ dynes per unit.

Distance from Centre.	Readings for Deflection.	Mean Deflection.	tan δ.	F $= .18 \tan \delta$.
10 cm.	$10\frac{1}{2}$, $10°$	$10\frac{1}{4}$	$\cdot 1854$	$\cdot 18 \times \cdot 1854$

The student should take a series of readings at different distances and enter the results in a table as in the case above, and then plot a curve of

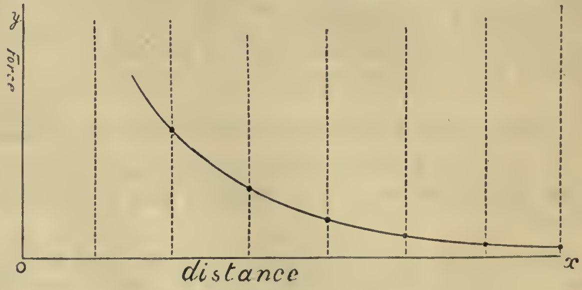


Fig. 32.

the results. Take a line on the squared paper to represent distances from the centre of the magnet, and draw perpendicular lines to represent the intensities at the given points.

EXPT. 2.—To compare the Strengths of two Poles by comparing the Forces they exert at a Point when placed at the same Distance from the Point.

Draw a line in the meridian, and place one of the magnets with one pole N in the same horizontal plane and in same meridian with the needle of a compass-box, and with the end S vertically above the centre of the box, and take the readings for deflection.

Repeat this with the south pole in position of N. Repeat the experiment with the other magnet.

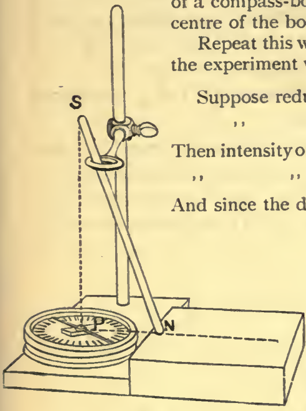


Fig. 33.

Suppose reduced readings are: 15° for first north pole.
,, ,, ,, 25° ,, second ,,

Then intensity of force at A due to first pole = H tan 30°.
,, ,, ,, second N pole = H tan 15°.

And since the distances are the same

$$\frac{m}{m'}=\frac{\text{intensity due to 1st pole}}{\text{intensity due to 2nd pole}}.$$

$$\therefore\ \frac{m}{m'}=\frac{\text{H}\tan 30^\circ}{\text{H}\tan 15^\circ}=\frac{\tan 30^\circ}{\tan 15^\circ}.$$

EXPT. 3.—To find the Force at Points along a Line bisecting the Axis of a Magnet at Right Angles.

Place a bar magnet perpendicular to the magnetic meridian, and draw a line through its centre in the meridian. Find approximately the distances from the centre of the magnet when the intensity due to the magnet is 2, 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ of the earth's horizontal intensity.

Note.—In the next section it will be shown that this intensity varies inversely as the cube of the distance from N or S. The student may verify this here. Find NS; measure NP in each case; find NP^3; and the various value of NP^3 will be found to be approximately in ratio, $\frac{1}{2}$, 1, 2, 3, 4.

5. Recapitulation.

← NB

1. Definition of **poles** of a magnet see Sect. III. Solenoidal magnet.

2. **Equality of Magnetic Poles.**—Two poles are said to be equal if they exert equal forces on a given north pole when placed at equal distances from it.

3. Definition of **intensity** at a point of a magnetic field: viz. the force exerted on the unit north pole placed at the point.

4. The force exerted on a pole of m units placed at a point where the intensity is H dynes-per-unit is Hm dynes.

5. Definition of **magnetic moment**—viz. the couple required to hold the magnet at right angles to a field of unit intensity.

6. The **couple** exerted by a parallel magnetic field of intensity H dynes per unit on a magnet of magnetic moment M units, when the axis of the magnet makes an angle δ with the field is $HM \sin \delta$.

7. The proof that the magnetic moment (M) of a solenoidal magnet (m, $2l$) is given by $M = 2ml$.

8. If at any point the intensity (R dynes per unit) is due to two parallel fields at right angles to each other, the intensities of which are respectively H dynes per unit, and F dynes per unit—then if one of these intensities, say H, is known the other two are given by the equations

$$F = H \tan \delta$$
$$R = H \sec \delta$$

where δ is the angle the resultant intensity R makes with the known intensity H.

EXERCISES

1. Define intensity at a point of a magnetic field. What are the forces acting upon a magnet placed in a parallel magnet field? A magnet whose axis is 20 cm., and strength of pole 30 units, is placed in a field of intensity 2 dynes per unit, write down the couple acting upon it when its axis makes angles of (i) 10°, (ii) 30°, (iii) 70°, (iv) 80° with the field.

2. A magnet ($2l = 40$ cm., $m = 80$) is placed with its axis at right angles to the direction of a field of intensity 10,000 dynes per unit. It is capable of turning about a vertical axis; what force in grammes weight must be applied at the circumference of the axes of rotation at a distance of 1 inch from the centre to keep it in position (1 inch $= 2\frac{1}{2}$ cm.)?

3. A pulley 30 cm. diameter is capable of turning freely about a vertical axis, and it has a magnet fixed on it parallel to its plane. The magnetic moment of the magnet is 6000 units, and the intensity of the field is 10,000 dynes per unit. A cord passes round the pulley, find the force which must be exerted along this cord to hold the magnet (i) at right angles, (ii) at an angle of 30°, (iii) 10°, to the direction of the field.

4. Define magnetic moment of a magnet, and prove that the moment of a solenoidal magnet is proportional to the strength of its poles and length of its axis.

5. A magnet is placed with its axis perpendicular to the magnetic meridian, and a magnetometer is moved along its axis so that the force upon it due to the magnet is always perpendicular to the meridian.

Write down the intensity due to the magnet at the points where the magnetised needle is observed to set at angles 10°, 20°, 30°, 40°, with the meridian [$H = \cdot 18$]. Write down also the intensity of the field at these points due to the earth and the magnet.

SECTION V

On the Vibration of a Magnet placed in a Magnetic Field

1. **On the Vibration of a Magnet.**—If a magnet be suspended in a stirrup by a silk fibre, it will oscillate about its position of rest, and the time of a single vibration can be determined by counting the number of vibrations in a given time, say five minutes; for it can be shown, theoretically and practically, that the time of each vibration is the same, provided that the magnet does not at any time oscillate through too great an angle. In fact, the vibration of a magnet is much the same as the vibration of a pendulum.

The time of vibration of the magnet depends upon the following conditions:—

(i) If the forces acting upon the magnet be increased, the time of vibration will be diminished, and therefore the magnet will oscillate quicker. Now the forces on the magnet may be increased in two ways:—

(*a*) By increasing the magnetic moment of the magnet.

(*b*) By increasing the strength of the field of force in which the magnet is oscillating.

(ii) If the mass and shape of the magnet be altered, the time of vibration will alter, just as the time of vibration of a pendulum is decreased as the length of the pendulum is shortened. The greater the mass of the magnet, and the more this mass is concentrated towards the ends, the longer the time of vibration becomes, and therefore the more slowly the magnet vibrates. This can easily be shown by placing round each end of the magnet a ring of india-rubber. This will increase the mass and change the shape of the magnet without altering the magnetic moment or field of force. It will be found that the magnet vibrates more slowly.

These facts respecting the time of vibration of a magnet placed in a magnet field are expressed in the following equation, which gives the number (n) of oscillations per second made by a magnet of moment M oscillating in a field of intensity H. The equation is—

$$\boxed{HM = 4\pi^2 n^2 I.}$$

Here $\pi = 3{\cdot}1416$, and I is the moment of inertia of the magnet,—a quantity which depends upon the mass of the body and upon the distance of the parts of the mass from the axis of rotation. From this equation it is seen that the number of vibrations is increased by increasing H and M, and decreasing I.

The following experiments will illustrate these facts :—

EXPT. 1.—To verify that the Number of Vibrations of a Magnet is doubled if the Intensity of the Field is made four times as great.

Place a bar magnet perpendicular to the magnetic meridian at NS, and in the axis of it produced place the magnetometer at a distance 30 cm. approximately; read the deflection in the usual way.

Now replace the compass-box by a short magnet suspended by a fine fibre, and count the number of oscillations in five minutes, and find the number of oscillations per minute.

Now replace the compass-box, and bring up the magnet until the secant of the deflection (see Note, page 38) is made four times as great as before. Then the intensity at the centre will be made four times as great. Now replace the compass-box by the oscillating magnet, and count again the number of oscillations in five minutes.

This number will be found twice as great as before.

EXPT. 2.—To show that the Time of Vibration of a Magnet depends upon the Mass and Shape of the Vibrating Body.

Magnetise a steel knitting-needle, and suspend it by a silk fibre. Take two small cylindrical pieces of lead, with holes along their axis so that the magnet can be passed through them. By this means the moment of inertia of the vibrating body can be altered by sliding the piece of lead near or farther from the axis of vibration, or by taking them off altogether. Count the number of vibrations when the sliders are (i) off, (ii) at the ends, (iii) half-way to the axis, (iv) close up to the axis.

EXPT. 3.—To show that the Time of Vibration depends upon the Magnetic Moment of the Magnet.

Take a bundle of steel knitting-needles and suspend it by a fibre, and count the number of oscillations in five minutes. If it is not magnetised it will not oscillate about the meridian; if it is slightly magnetised it will oscillate very slowly.

Now magnetise one of the knitting needles and find time of vibration; then magnetise another of the needles and find time of vibration.

2. **Intensities at Points of a Magnetic Field.**—From the relation $HM = 4\pi^2n^2I$ we see that **the intensity at a point of a field varies as the square of the number of vibrations a small magnet makes at the point.**

This gives a ready method of comparing the intensities at different points of a field: all that is necessary is to find the number of oscillations of a small magnet at the specified points of the field.

The oscillations should be counted for five minutes, and the magnet should vibrate through *small* arcs.

EXPT. 1.—**To compare the Intensities at the five given Points.**

These may be five points on the tables, and the fields may be created by arrangement of magnets and soft iron.

A convenient way of expressing the results is to express the intensities in terms of the intensity due to the earth.

Example:—

Number of oscillations in five minutes 120 due to intensity at first point.
,, ,, ,, ,, 80 ,, of earth alone.

$$\therefore \quad \frac{F'}{H} = \frac{4\pi^2n_1^2I}{4\pi^2n^2I} = \frac{n_1^2}{n^2} = \frac{(120)^2}{(80)^2} = \frac{9}{4}$$

$$\therefore \quad F' = 2{\cdot}25H,$$

and so for the others; enter results in tabular form.

In some important cases the field of force is due to the earth and to a magnet which exerts its force in the magnetic meridian either with or against the earth.

In this case it is easy to find the intensity due to the magnet alone in terms of the earth's intensity.

For example, suppose in Expt. 1 above, the first point was at a point P in the axis of a magnet NS placed in the magnetic meridian, with its N pole towards the north: Then if F is intensity at P due to magnet alone—

$$F' = F + H$$

$$\therefore \quad F + H = 2{\cdot}25\ H$$

or
$$\begin{aligned}F &= 2{\cdot}25\,\mathrm{H} - \mathrm{H}\\ &= 1{\cdot}25\,\mathrm{H}.\end{aligned}$$

If the magnet had, however, been placed in the *opposite direction*, then the force F′ would be given by

$$\begin{aligned}F' &= F - H\\ \therefore\quad F - H &= 2{\cdot}25\,H\\ F &= 3{\cdot}25\,H.\end{aligned}$$

In the general case, if n is number of vibration in a given time due to earth alone, n' due to the bar magnet and earth at same point,

$$F' = \frac{n'^2}{n^2}\,.\,H;$$

and in first case when the magnet and earth act together,

$$\begin{aligned}F' &= F + H\\ \therefore\quad F + H &= \frac{n'^2}{n^2}H,\end{aligned}$$

or

$$F = \frac{n'^2}{n^2}H - H = \frac{n'^2 - n^2}{n^2}H.$$

In the second case, when the magnet and earth act against each other—

$$\begin{aligned}F - H &= \frac{n'^2}{n^2}H\\ \therefore\quad F &= \frac{n'^2}{n^2}H + H = \frac{n'^2 + n^2}{n^2}H.\end{aligned}$$

If it is desired to find the intensity at any point in the axis of a magnet due to the magnet alone, the magnet should be placed as in the first case, because the equation is

$$F = \frac{n'^2 - n^2}{n^2}H = \frac{(n' - n)(n' + n)}{n^2}H.$$

i.e. suitable for logarithmic calculation.

EXPT. 2.—Find by Method of Oscillations the Intensity at Points along the Axis of a Bar Magnet.

Place the magnet with its axis in the meridian with its north pole towards the north.

Let n', n'', n''', etc., be the number of vibrations made per second at the different points, say, 5, 10, 15 cm. from one pole of the magnet, and n the number of vibrations per second in the earth's field only.

Then

$$F' : F'' : F''' : \ldots$$
$$:: \frac{n'^2 - n^2}{n^2} : \frac{n''^2 - n^2}{n^2} : \frac{n'''^2 - n^2}{n^2} \ldots$$

EX. 1.—A magnet makes 20 oscillations per minute in the earth's field of force, and 8 per minute at a point of another field. Compare the two intensities.

$$\frac{F}{H} = \frac{8^2}{20^2} = \frac{4}{25}$$
$$\therefore \quad F = \frac{4}{25} . H$$

If $H = \cdot 18$ dynes per unit
then $F = \cdot 0288$ dynes per unit.

EX. 2.—A magnet makes 40 oscillations per five minutes with earth's force alone, but under action of a magnet and the earth (both in same direction) it makes 50 in five minutes: Find (1) the intensity due to magnet alone; (2) the number of oscillations in five minutes the magnet will make if the bar magnet has its poles reversed.

$$\frac{F + H}{H} = \frac{50^2}{40^2} = \frac{25}{16}$$
$$\therefore \quad F + H = \frac{25}{16} H$$
$$F = \frac{9}{16} H.$$

If this intensity is reversed, and the number of oscillations in five minutes is n, then

$$\frac{-F + H}{H} = \frac{n^2}{40^2}$$
$$\therefore \quad \frac{n^2}{40^2} = \frac{H - \frac{9}{16}H}{H} = \frac{7}{16}$$
$$n^2 = \frac{7}{16} \cdot 40^2$$
$$n = 40 \sqrt{\frac{7}{16}} = 26 \cdot 4$$

EX. 3.—Two magnetic poles are compared by counting the number of oscillations made by a small oscillating magnet when placed at the same distance from them—the pole and the magnet being in the magnetic meridian. [Arranged as in Fig. 33, except that the compass-box is replaced by the oscillating magnet.] Compare their strengths.

Let n be number of oscillations in a given time due to earth.
n' ,, ,, due to first pole and the earth.
n'' ,, ,, due to second pole and the earth.

Then if F′ is intensity due to first pole
F″ ,, ,, second pole.

$$\frac{F'+H}{H}=\frac{n'^2}{n^2}$$

or,

$$F'=\frac{n'^2-n^2}{n^2}H.$$

Similarly,

$$F''=\frac{n''^2-n^2}{n^2}H$$

$$\therefore\quad \frac{F'}{F''}=\frac{n'^2-n^2}{n''^2-n^2}.$$

If m' is strength of first pole, m'' of second, then since the distances are the same.

$$\frac{F'}{F''}=\frac{m'}{m''}$$

$$\therefore\quad \frac{m'}{m''}=\frac{n'^2-n^2}{n''^2-n^2}.$$

EXERCISES

1. Upon what does the time of vibration of a magnet depend? Describe experiments to show the effect of each quantity.

2. A magnet is found to vibrate 10 times per minute: how would you adjust it so as to vibrate 5 times per minute in the same field.

3. You are given a piece of steel to make into a magnet. How would you ascertain when it has reached its maximum strength? Without altering its strength how could you make it vibrate more slowly? Describe a magnet the time of vibration of which can be varied.

4. A magnet vibrates 10 and 12 times per minute respectively at two points of a field. Compare the intensities at these two points.

5. A magnetic needle makes 12 oscillations a minute under the earth's force alone. Under the influence of the earth and a magnet A it makes 14 oscillations, and 16 oscillations when A is replaced by another magnet B. Compare the magnetic intensities at the point due to A and B.

6. The magnetic moment of a magnet is 300 units, and it vibrates 10

times per minute. It is magnetised until it vibrates 15 times per minute in the same field. What is its new magnetic moment?

7. A needle under the earth's influence alone vibrates 6 times per minute, when a certain magnet is held near it the number of oscillations are 7 per minute; and when the magnet is replaced by another the number is increased to 8 per minute. Compare the magnetic intensities at the point due to the two magnets.

8. Show how to compare the intensities at points of a field by the method of oscillation.

A small magnet is placed at five points in a field of force. The number of oscillations in five minutes are counted and are found to be 50, 55, 30, 80, 75 respectively. It is known that the intensity at the first point is ·18 dynes per unit: find the intensities at the other points.

9. Show how to compare magnetic moments of magnets by oscillation methods.

Two cylindrical magnets, whose moments of inertia are equal, are suspended in the same field, and it is found that they make respectively 10 and 12 vibrations in two minutes. Compare their magnetic moments.

10. A cylindrical magnet, 10 cm. long, 5 cm. thick, of mass 1500 gms., is found to make 56 vibrations in five minutes in the earth's field $(H = \cdot 18)$: find its magnetic moment; and if the length of its axis is 9 cm., find the strength of its poles.

11. A bar magnet is placed with its axis in the magnetic meridian, with its north pole towards the south, and at two points in the axial line a magnet is found to make 50 and 30 vibrations per minute. The same magnet makes 40 vibrations per minute under the action of the earth's force alone. Find the intensity at these two points due to the bar magnet.

12. A small magnet is oscillating under the influence of the earth's force alone. How would you arrange a bar magnet horizontally with its centre vertically above this magnet in order to (i) increase the rate of oscillation, (ii) diminish the rate, (iii) reduce the oscillation to zero?

SECTION VI

On the Laws of Force at Points in a Magnetic Field

1. **On the Force between two Magnetic Poles.**—By the method given above the intensity of the magnetic force at points in a magnetic field can be determined. It remains now to determine the law connecting the intensity with the position of the point in the field and the cause of the field. There are two methods of investigating these relations.

(1) The first method is due to Coulomb. Coulomb discovered by means of his torsion balance that the force between two magnetic poles varies inversely as the square of the distance between them. Thus, if we assume that the force on a quantity

of magnetism due to one quantity is not affected by another quantity, and if we know the distribution of magnetism, then the force at any point can be determined by mathematical means. Examples of this method will be given below. It has been very powerful in the hands of mathematicians.

The following are the laws of magnetic force between two quantities of magnetism supposed to be concentrated at points. They are frequently called the "**Laws of Magnetic Force**"—

i. Poles of like magnetism are repelled from each other; of unlike magnetism are attracted to each other.

ii. The force between two quantities of magnetism is proportional to the quantities of magnetism.

iii. The force between two given quantities of magnetism varies inversely as the square of the distance between them.

The first law has been illustrated in Section I.; the second is not strictly a law, but a definition of strength of magnetic pole (see § III).

Law iii. This law can be verified by the torsion balance. All that is necessary is to find the force between the poles at different distances apart. For example, suppose at distances of 1, 2, 3, 4, 5, 6 cm. apart, the angles of twist are 72°, 18°, 8°, $4\frac{1}{2}$°, 2·8°, 2°. The force at distance of

1 cm. $= k \times 72$
at 2 cm. $= k \times 18$
at 3 cm. $= k \times 8$,
and so on.

Careful experiments performed by Coulomb showed that the force between two poles varies inversely as the square of the distance between the poles.

The unit quantity of magnetism is that quantity which, placed at a distance of 1 cm. from a similar equal quantity, is repelled from it with a force of 1 dyne.

Hence if m, m' be the strength of two poles at a distance r cm. apart, the force between them is given by

$$\boxed{\mathrm{P}=\frac{m.m'}{r^2}\text{ dynes.}}$$

(2) The second method is due to Faraday. If a surface be taken which is such that its boundary is formed by lines of force, such a tubular

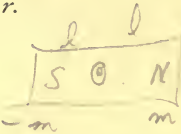
surface is called a tube of force. Faraday proved that the intensity at different points of a small tube of force is inversely proportional to the cross-sectional area of the tube. This, by magnetic methods only, is difficult to verify with any degree of accuracy, but it is a useful exercise for a student to attempt it. Place a flat bar magnet on a sheet of paper, and draw carefully two lines of force near each other. Now find the number of oscillations at different points between these lines, and thus find the intensity R, say $\left(\text{for } R=\frac{n'^2}{n^2}H\right)$, and measure carefully the cross distance (τ') between the lines at the point, and take the cross sectional areas to be proportional to the square of these distances. Since R is proportional to n'^2 and the sectional area of τ'^2, then $n'\tau'$ should be found constant.

If the tube of force is conical, then since the cross sectional area varies as the square of the distance from the vertex, it follows that the intensity varies inversely as the square of the distance from the vertex, *i.e.* from the pole to which the straight lines converge. We thus see that the "law of inverse squares" is a consequence of Faraday's law.

2. On the Intensities at Points in the Field of Force due to a Solenoidal Magnet.

PROP. 1.—*To find the intensity at a point in the axial line of a magnet.*

Let NS be the axis of the magnet of length $2l$ cm. Let m be the strength of the poles. Let O be the centre of the magnet. Let P be the point such that $OP=r$.



Then

$$NP=r+l$$
$$SP=r-l.$$

Intensity at P due to m units at N

$$=\frac{m}{NP^2}$$

$$=\frac{m}{(r+l)^2} \text{ along OP.}$$

Intensity at P due to m units at S

$$=\frac{m}{SP^2}$$

$$=\frac{m}{(r-l)^2} \text{ along PO.}$$

∴ Intensity at P due to both quantities at S and N

$$=\frac{m}{(r-l)^2}-\frac{m}{(r+l)^2} \text{ along PO}$$

$$= m\left\{\frac{(r+l)^2-(r-l)^2}{(r-l)^2(r+l)^2}\right\}$$
$$=\frac{m\,.\,4rl}{(r^2-l^2)^2}.$$

Now M the magnetic moment of the magnet is given by

$$M = 2ml$$
$$\therefore\quad F = \frac{2Mr}{(r^2-l^2)^2}.$$

If r is so great that l^2 may be neglected compared with r^2, then

$$F = \frac{2Mr}{r^4} = \frac{2M}{r^3}.$$

PROP. 2.—*To find the intensity of the field due to a solenoidal magnet at points in a plane drawn through the centre of the axis at right angles to it*, i.e. *at points in the equatorial plane of the magnet.*

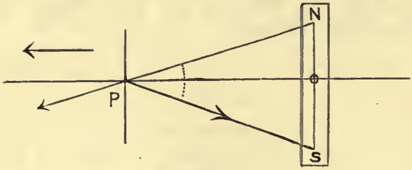


Fig. 34.

Let NS be the axis of length $2l$, m the strength of the poles

$$OP = r.$$

Intensity due to m at N is $\frac{m}{NP^2}$ along NP.

Intensity due to m at S is $\frac{m}{SP^2}$ along PS.

And since these are equal (NP = SP) their resultant will bisect the angle between their directions, and will therefore be along a line parallel to the axis NS.

Now resolving each intensity parallel to the axis:

If θ = angle PNO = angle PSO

resolved part of $\frac{m}{NP^2}$

$$=\frac{m}{NP^2}\cos\theta, \text{ parallel to the axis}$$

and resolved part of $\frac{m}{SP^2}$

$$=\frac{m}{\mathrm{SP}^2}\cos\theta, \text{ parallel to the axis.}$$

Hence intensity parallel to the axis

$$=\frac{m}{\mathrm{NP}^2}\cos\theta+\frac{m}{\mathrm{SP}^2}\cos\theta$$

$$=\frac{2m}{\mathrm{NP}^2}\cos\theta.$$

$$\text{If NP}=\rho, \text{ then } \cos\theta=\frac{l}{\rho}$$

and intensity at $\mathrm{P}=\frac{2ml}{\rho^3}=\frac{\mathrm{M}}{\rho^3}$,

$$\therefore\quad \mathrm{F}=\frac{\mathrm{M}}{\rho^3}.$$

If the length PO is great compared with ON, then ρ is nearly equal to OP, *i.e.* to r.

$$\therefore\quad \mathrm{F}=\frac{\mathrm{M}}{r^3}.$$

3. To compare the Intensities at Points in the Axial Line or Equatorial Plane of a Magnet with the Earth's Horizontal Intensity.

The intensities can be compared with the earth's intensity by placing the magnets at right angles to the meridian, and placing a compass-box at a point either

(1) in the axial line,
(2) in an equatorial line.

Since in each case the intensity F is perpendicular to the intensity of the earth's horizontal force H, then if δ is the angle the compass-needle makes with the meridian, in each case

$$\mathrm{F}=\mathrm{H}\tan\delta.$$

But in case i.

$$\mathrm{F}=\frac{2\mathrm{M}r}{(r^2-l^2)^2}$$

$$\therefore\quad \frac{2\mathrm{M}r}{(r^2-l^2)^2}=\mathrm{H}\tan\delta,$$

or if l^2 be small compared with r^2

$$\frac{2\mathrm{M}}{r^3}=\mathrm{H}\tan\delta.$$

In case ii.

$$F=\frac{M}{\rho^3},$$

$$\therefore\quad \frac{M}{\rho^3}=H\tan\delta.$$

$$\therefore\quad M=H\rho^3\tan\delta,$$

or if ρ is great compared with l

$$M=Hr^3\tan\delta.$$

These relations between M and H furnish additional methods of comparing magnetic moments, and of determining them absolutely. They were first used by Gauss, and the two positions of the magnetometer, *i.e.* in the equatorial or in the axial line of the magnet, are known as Gauss's A and B positions respectively.

In Gauss's A position, *i.e.* where the magnetometer is in the equatorial line,

$$M=H\rho^3\tan\delta.$$

In Gauss's B position, *i.e.* where the magnetometer is in the axial line,

$$M=\frac{H(r^2-l^2)^2}{2r}\tan\delta,$$

or in the special case of a short magnet such that l^2 is small compared with r^2,

$$M=\frac{Hr^3}{2}\tan\delta.$$

EXERCISES

1. What quantity of magnetism is taken as the unit? Why is this unit called the centimetre-gramme-second (C.G.S.) unit? If the unit of magnetism were taken as that quantity which repels an equal quantity placed at a distance of 1 foot with the force of a grain weight, how many times would this unit contain the C.G.S. unit?

2. State the Three Laws of Magnetic Force. If one pole repels another at a distance of 1 cm. with a force of 10 dynes, with what force will they repel each other at a distance of 5 cm.?

3. Find the force between two units at a distance of 1 foot (30·48 cm.) from each other.

4. The force between a pole of 50 units and another pole is 350 dynes at a distance of 1 cm. Find (i) the strength of second pole, (ii) the force between the two poles at a distance of 8 cm.

5. A south pole of 80 units, and a north pole of 80 units, are placed at a distance of 15 cm. apart. In the same line with them, and at a distance of 30 cm. from the north pole, is placed a north pole of 50 units.

Find (i) Force on the last pole due to the north pole;
(ii) Force on the same pole due to the south pole;
(iii) Force on the same pole due to both the other poles.

6. Find the force between the poles in the following cases:
(i) 500 units and 319 units at distance of 2 mm.;
(ii) 1350 units and 1191 units at distance of 3 metres;
(iii) 300 units and 20 units at distance of 3 feet.

7. Find the number of dynes in a grain weight. Find the quantity of magnetism which will repel a like quantity of magnetism placed at a distance of 30·48 cm (a foot) with a force of a grain weight.

8. Find the quantity of magnetism which, placed at a distance of 30 cm. from an equal quantity, will repel it with a force of 981 dynes.

9. Two magnetic poles repel each other with a force of 300 dynes when placed 10 cm. apart. Find the distance between them when the force is 450 dynes.

10. Two magnetic poles repel each other with a force of 550 dynes. Find the distance between them when the force is (i) doubled, (ii) trebled.

11. What is the magnetic moment of a magnet, the strengths of the poles of which are respectively 20 units, and the length of the axis 3 cm.?

12. Find the magnetic moment of a magnet, the poles of which are 1 foot apart, and of unit foot-grain-second strength.

13. What is meant by magnetic strength or intensity at a point of a field of force?

If the magnetic strength at a given point is ·2 dyne per unit quantity of magnetism, what is the force acting on 200 units of magnetism?

14. What is meant by the earth's horizontal magnetic intensity? Where does it vanish on the earth's surface, and where is it greatest?

15. The force at a point of a field is ·18 dyne per unit quantity of magnetism. What is the force on the unit foot-grain-second quantity of magnetism?

16. Find the force at the centre of a magnetometer due to a magnet placed in one of Gauss's positions, the deflection being 10° 30′ (the value of the $H = \cdot 18$).

17. Two magnetic moments are compared by Gauss's deflection method. The deflections are 18° and 36° respectively. Compare the moments.

18. The magnetic moments of two magnets are respectively 1850 and 3250 units, and the first gives a deflection of 10° 30′. Find the deflections produced by (i) the second magnet, (ii) the two together side by side.

19. If the position in Question 18 is Gauss's A position, and the distance (r) is 30 cm., find the value of the forces; and if the length of each axis is 15 cm., find the strengths of the poles.

20. A magnet (axis 10 cm., strength of poles 2592 units) is placed with its axis in the magnetic meridian and its south pole towards the north. Show that the force at a distance of 35 cm. from the centre is such as to balance that of the earth's magnetic force, *i.e.* be equal to ·18 dyne per unit quantity of magnetism.

21. A bar magnet is placed with its axis in the magnetic meridian and its south pole towards the north. The zero point is at a distance of 45 cm. from the centre. Find the magnetic moment of the magnet.

22. Two north poles are placed at a distance of 30 cm. apart; the

point of zero intensity is 10 cm. from one of them. Compare the strengths of the poles.

23. How would you compare roughly the strengths of two poles by iron filings?

24. The force between the two poles of a magnet is ·5 dyne, and the axis is 10 cm. Find the strength of the poles and the magnetic moment.

25. Find the force between two poles of 150 and 250 units strength, the distance between which is 15 mm.

26. At what distance must a pole of 100 units be placed from one of 125 units to produce a force of 130 dynes.

27. A magnet has an axis of 5·5 cm., and the strength of each of its poles is 100 units. Find the force on a unit of magnetism at a point in the line of the axis 5 cm. from the centre of the magnet.

SECTION VII

On Magnetic Measurements

1. The quantities to be measured in terms of fundamental units or to be compared with each other are magnetic moments, strength of poles, axes of magnets, intensities at points of a field.

The methods adopted are—

(1) *The Statical Methods.*

i. Balancing couples due to a magnetic field against the couple due to the torsion of a wire, *i.e.* using the torsion balance.

Let

k = reduction factor of the magnetic torsion balance.
θ = angle the wire is twisted through.
$2l$ = length of axis of suspended magnet.
m = strength of pole of suspended magnet.
H = intensity of field at the pole.

Then

$$\boxed{Hm \,.\, 2l = k\theta.}$$

This is the fundamental equation of the torsion balance.

(ii.) Balancing one magnetic couple against another, *i.e.* using the compass-box or magnetometer.

It is always convenient to arrange that the magnetic forces of one couple shall be **perpendicular** to the magnetic forces of the other—in this case if F and H are the intensities of the two fields respectively, and δ the angle the compass-magnet makes with the direction of H, then

$$F = H \tan \delta.$$

(2) *The Dynamical Method.*—This method enables us to measure couples due to magnetic fields by means of oscillation —for the time of vibration of a body depends upon the couple acting upon it.

Let

M = magnetic moment of a magnet.
I = the moment of inertia of the magnet.
H = intensity of the field in which the magnet oscillates.
n = number of oscillations per second.

Then

$$HM = 4\pi^2 n^2 I.$$

Note.—For a cylindrical magnet vibrating about an axis through the centre and perpendicular to its own axis

$$I = m\left\{\frac{l^2}{12} + \frac{a^2}{16}\right\}$$

where in this case

m = mass of the magnet (not to be confused with strength of pole).
l = length of magnet (not to be confused with $2l$ = length of *axis* of magnet).
a = thickness of magnet.

For an oblong magnet of length l and horizontal breadth b, and mass m, vibrating about an axis through its centre perpendicular to the face containing l and b

$$I = m\left\{\frac{l^2 + b^2}{12}\right\}$$

It is important to notice that methods by the torsion balance and by oscillation can be applied when the magnets have *no* poles, *i.e.* are not necessarily solenoidal magnets.

A.—To compare Magnetic Moments

(i) *By Magnetometer.*—Arrange one of the magnets with its axis perpendicular to the meridian, and so that the centre of the magnet of the magnetometer shall be at a point either in the axial line of the magnet or in the equatorial line, and take readings for the deflection δ.

Now place the second magnet in the same relative position to the magnetometer, with its centre in the same position as the first magnet, and find the deflection δ.

If the position is Gauss's A position (equatorial) then

$$F=\frac{M}{\rho^3}$$

$$\therefore \quad H\tan\delta=\frac{M}{\rho^3}$$

$$\therefore \quad M=H\rho^3\tan\delta \qquad \text{for 1st magnet,}$$

also

$$M'=H\rho^3\tan\delta' \qquad \text{for 2nd magnet,}$$

$$\therefore \quad \frac{M}{M'}=\frac{\tan\delta}{\tan\delta'}\,.$$

If the position is Gauss's B position (axial) then

$$F=\frac{2M}{r^3} \qquad \left(\text{neglecting } \frac{l^2}{r^2}\right)$$

$$M=\frac{Hr^3}{2}\tan\delta \qquad \text{for 1st magnet,}$$

$$M'=\frac{Hr^3}{2}\tan\delta' \qquad \text{for 2nd magnet,}$$

$$\therefore \quad \frac{M}{M'}=\frac{\tan\delta}{\tan\delta'}\,.$$

Note.—If l cannot be neglected then the more complex formulæ must be used.

(ii) *By Oscillations.*—Suspend the first magnet in light paper stirrup by a few fine silk fibres—within an inverted bell jar or a square box with a glass front—and count the number of vibrations in, say, five minutes.

In the same way find the number of vibrations of the second magnet in five minutes.

Now for 1st magnet

$$HM=4\pi^2n^2I$$

where n = number of vibrations per second.

For 2nd magnet

$$HM'=4\pi^2n'^2I'$$

$$\therefore \quad \frac{M}{M'}=\frac{n^2 I'}{n'^2 I}\,.$$

If $I=I'$, then

$$\frac{M}{M'}=\frac{n^2}{n'^2}\,.$$

Note.—This method does not depend upon the form of the magnet, as to whether it has or has not geometrical poles. It can therefore be used to compare the magnet moments of any magnets, provided that their moments of inertia are equal or can be found either by direct measurement or by experiment.

B.—To compare the Strengths of Poles

(i) *By Torsion Balance.*—Suppose it is required to compare the strength of the poles of two long thin magnets.

Place the pole of one of the magnets at a suitable distance from the pole of the torsion balance magnet, and find the angle θ for equilibrium.

Now place the second pole at the same distance, and find the angle of torsion θ'.

If H, H′ are the intensities at the pole of the torsion magnet due respectively to the two poles, then

$$\frac{H}{H'}=\frac{\theta}{\theta'}$$

But

$$H=\frac{m}{r^2}$$

and

$$H'=\frac{m'}{r^2},$$

where m, m' are the strength of the poles and r is the distance each pole is placed from the torsion magnet pole.

$$\therefore\ \frac{H}{H'}=\frac{m}{m'},$$

$$\therefore\ \frac{m}{m'}=\frac{\theta}{\theta'}.$$

(ii) The strength of the poles can also be compared if the magnetic moments M and M′ are compared by methods in A, and the axes $2l$, $2l'$ found as in Expt. 1, § II.

For

$$M=2ml$$

$$M'=2m'l'$$

$$\therefore\ \frac{m}{m'}=\frac{Ml'}{M'l}$$

C.—To compare the Intensities at Points of a Magnetic Field.

This is done by the methods of oscillation. See § V.

D.—Given the Intensity of a parallel Field, to find the Magnet Moment, the Axis, and the Strength of the Poles of a Magnet.

(i) *By Torsion Balance.*—Suspend the magnet in the torsion balance, and find the torsion required to hold it at right angles to the field of intensity H. Then,

$$Hm \,.\, 2l = k\theta,$$

i.e.

$$HM = k\theta,$$

$$\therefore \quad M = \frac{k\theta}{H}.$$

(ii) *By Magnetometer.*—Find the deflection δ as in A (i), and use the equations—

$$\text{Gauss A.} \quad M = H\rho^3 \tan\delta$$

$$\text{Gauss B.} \quad M = \frac{Hr^3}{2}\tan\delta,$$

or the more complex equations if necessary.

(iii) *By Oscillations.*—Find the number of oscillations made by the magnet as in A (ii), and use the equation—

$$HM = 4\pi^2 n^2 I$$

$$\therefore \quad M = \frac{4\pi^2 n^2 I}{H}$$

I must be determined by direct measurements, or by experimental methods.

E.—To find the Magnetic Moment of a Magnet and the Intensity of a Parallel Field absolutely.

This method depends upon two separate experiments of A (i), A (ii).

We have the two equations—

$$HM = 4\pi^2 n^2 I$$

$$\frac{M}{H} = \rho^3 \tan\delta.$$

Now by Expt. A (ii) HM can be found.

And by Expt. A (i) $\frac{M}{H}$ can be found.

Hence multiplying these together we get

$$M^2 = 4\pi^2 n^2 I \rho^3 \tan\delta.$$
$$\therefore\quad M = 2\pi n\sqrt{I\rho^3 \tan\delta}.$$

And dividing the first by the second we get—

$$H^2 = \frac{4\pi^2 n^2 I}{\rho^3 \tan\delta},$$

or

$$H = 2\pi n\sqrt{\frac{I}{\rho^3 \tan\delta}}.$$

If now the axis be measured and found to be $2l$, m can be found absolutely.

F.—To determine completely the Intensity at a Point of the Earth's Magnetic Field.

To completely determine the intensity of the earth's field at a point, we must find—

(1) The magnitude of the force on a unit.
(2) The direction referred to the geographic meridian and to the vertical.

If F is the resultant of the earth's intensity at a point, δ the dip at that point, then

$$H = F\cos\delta$$
$$\therefore\quad F = H\sec\delta.$$

To determine F we must therefore find H and δ. H may be found as in E, and δ is found by means of a dip circle.

[*Note.*—Thevertical intensity can be found from the relation $\frac{H}{V} = \text{cotan}\,\delta$, *i.e.* $V = H\tan\delta$.]

The direction of the intensity makes an angle δ with the horizontal plane; it is now necessary to find the angle that the plane through the vertical and through the direction of the intensity makes with the geographic meridian, *i.e.* to find the declination.

To recapitulate, in order to completely determine the earth's magnetic intensity we must find—

(1) The dip.
(2) The declination.
(3) Magnitude of intensity F (from $F = H \sec \delta$).

Exercises on Chapter I

I

1. What is meant by induced magnetisation? Two bar magnets are to be placed in a vertical position. What should be the relative position of their poles in order that they may attract an armature with the greatest possible force? What substance would be most advantageous to use for the stand on which their bases rest?

2. The beam of a balance is made of soft iron. When it is placed at right angles to the magnetic meridian, two equal weights placed in the opposite pans just balance. Will the weights still appear to be equal when the balance is turned so that the beam swings in the magnetic meridian? Give reasons for your answer.

3. Two bar magnets, the moments of which are in the ratio of 8 : 27 are placed with their centres 3 feet apart, their magnetic axes being in the same straight line, which is perpendicular to the magnetic meridian. If their north poles are turned towards each other, find the position which a small compass must occupy in the line joining the magnets, in order that it may point in the same direction as if the magnets were not there.

4. A glass tube containing four similar pieces of hard steel which just fill it when placed end to end, is suspended so that it can oscillate about its central point in a horizontal plane. What will be the nature of the difference (if any) in the times of oscillation when (1) the two outer pieces only, (2) the two inner pieces only, are magnetised, unlike poles being in both cases nearest together? Neglect the effects of induction, and give reasons for your answer. What, in each case, would be the effect of slight induction on the time of oscillation?

5. Explain why, in determining the magnetic dip at any place, it is necessary to reverse the magnetism of the needle, so as to make each end of it dip in turn.

6. Three magnets of equal strength are placed vertically, and a horizontal sheet of glass is placed on their upper poles, and iron filings are sprinkled over the glass. Draw the lines of forces indicated by the filings if two of the poles are north and one south.

II

7. A piece of steel wire, bent so as to form two sides of a square, is magnetised in such a way that each of its free ends is a N pole, and the bend a S pole. When placed upon a cork floating in water how will it set?

8. A large soft iron rod lies on a table in the magnetic meridian, and a dipping needle is placed at some distance and at about the same level (1) due south, (2) due north of it. How will the magnitude of the angle of dip be affected in each case? (Neglect any inductive action between the needle and the bar.)

9. A steel wire 12 inches long is magnetised and broken into two parts, 8 and 4 inches long respectively. How do the two parts differ from each other magnetically? Would they if similarly placed with respect to a small compass-needle produce equal deflections. If the longer one produces a deflection of 15° when placed so that its force is perpendicular to the meridian, what deflection will the other produce?

10. A long bar magnet lies north and south with its north-seeking pole turned towards the south. A small compass-needle is supported at a point on the magnetic axis of the bar produced. What is the effect on the time of vibration of the needle of sliding the bar nearer to the needle?

11. Explain how the magnetic meridian may be determined by a dip circle.

12. Describe the behaviour of a dip needle as it is carried from the neighbourhood of the north to that of the south pole of the earth.

III

13. A long magnet and a piece of soft iron of the same size and shape are placed parallel to each other underneath a sheet of paper upon which iron filings are strewed. How will the filings arrange themselves?

14. A tall iron mast is situated a little in front of the compass in a wooden ship. Explain the nature of the compass error when the ship is sailing in an easterly direction (1) in the northern, (2) in the southern hemisphere.

15. A rod of iron when brought near to a compass-needle attracts one pole and repels the other. How will you ascertain whether its magnetism is permanent or is due to the temporary induction from the earth?

16. Given a magnet and the means of suspending it, how will you determine (1) the magnetic meridian, (2) in which direction *north* lies? It is assumed that you do not know which end of your magnet is a north and which a south pole.

17. A magnet is placed horizontally in the magnetic meridian due south of a compass-needle. How will its action on the latter be affected if (1) a plate of soft iron is interposed between the two, (2) a rod of soft iron is placed along the line which joins their centres? Give reasons.

18. A bar magnet is supported horizontally by a fine vertical wire. The magnet lies in the magnetic meridian when there is no torsion in the wire, and the top of the wire must be turned through 100° in order to deflect the magnet 15° from the meridian. The magnet is then removed, remagnetised and replaced, and now the upper end of the wire has to be twisted through 150° to produce the same deflection of the bar. Compare the moment of the bar in the two cases.

IV

19. A light wooden rod a foot long is balanced at its centre on a fine

pivot, so as to turn freely in a horizontal plane. If a magnetised sewing needle is stuck through one end of the rod horizontally and at right angles to the rod, and balanced by a small counterpoise at the other end, how will the rod set itself?

20. What are the chief precautions needed in order to get an accurate determination of the magnetic dip?

21. The lower end of a fine wire, which hangs vertically, is fastened to the middle of a straight steel magnet, so that the magnet is suspended horizontally by the wire. When the wire is without twist, the magnet comes to rest in the magnetic meridian, but when the upper end of the wire is turned once round, the magnet is deflected from the meridian through 30°; how much must the top of the wire be turned to make the magnet set at right angles to the meridian?

22. A bar of very soft iron is set vertically; how will its upper and lower ends respectively affect a compass-needle? Would the result be the same at all parts of the world as it is in this country? If not, state generally how it would differ at different places.

23. Two equal and equally magnetised bar magnets are fastened together at their centres at right angles to each other, so as to form an equal-armed cross. How will the cross set itself when balanced at the middle upon a point?

24. A small magnet hanging by a silk fibre makes 10 oscillations in a minute when acted upon by the earth's magnetic force alone. When two equal masses of soft iron are placed at equal distances from it, one to the north and the other to the south, the magnet makes more than 10 oscillations in a minute; but when the same pieces of soft iron are placed at equal distances east and west of the magnet, it makes less than 10 oscillations in a minute. Why is this? explain with diagram. If the oscillations are respectively 12 and 8 per minute and $H = \cdot 18$, find the intensity in each case.

V

25. What is meant by line of magnetic force? Define intensity at a point of a field. What is a uniform field of force. Show that if the lines of force in a magnetic field are all straight and parallel the field of force must be uniform.

26. What is meant by the axis of a magnet? Show how to determine the position within a magnet of its magnetic axis.

27. What is the condition that a piece of steel shall be uniformly magnetised? What do you understand by the "pole of a magnet"? Describe experiments with a torsion balance (i) to test the equality of the strengths of two magnetic poles, (ii) to prove that the two poles of a uniformly magnetised thin piece of steel are of equal strength.

28. You are given two long thin pieces of hard steel, show how to magnetise them so that the strength of the pole of one of them shall be three times the strength of the pole of the other.

29. A magnet is placed in a uniform magnetic field with its axis inclined to the lines of force; what are the forces acting upon? Investigate the magnitude of the couple formed by these forces.

30. Define magnetic moment of a magnet. Show how to find the time

of vibration of a magnet in a uniform magnetic field, when it is making small oscillations freely about its position of rest.

31. Describe the methods of investigating the intensity of the forces at points of a field (i) by means of torsion balance, (ii) by oscillations of a short magnet (why should the magnet be short?)

32. On what experiments is our knowledge of the law of force between two magnetic poles based?

Assuming the law of inverse squares, find the force exerted by a bar magnet on a unit pole placed at a point in the equatorial line. Show how to verify this by deflection methods.

33. Find the force exerted by a small bar magnet on a magnetic pole in the axis produced.

The centres of two small magnets coincide and their axes are at right angles, the magnetic moment of the one being twice that of the other. Show that the lines of force due to the combination are at all points in the axis of the second magnet produced inclined at 45° to that axis.

34. In expressing the strength of a magnetic pole what is the unit adopted?

State briefly the experimental methods of finding the magnetic strength of the pole of a magnet.

35. State briefly (i) the methods of comparing magnetic moments of magnets, and state their respective advantages.

(ii) Methods of comparing the strengths of poles of a magnet.

(iii) Methods of comparing intensities at points of a field.

36. Give in outline the methods of finding the magnetic moment and magnetic polar strength of a magnet in absolute measure, and the intensity at a point of a field of force.

37. What must be known to completely determine the intensity of a field of force at any point? State how each quantity may be experimentally determined. Take as an example the intensity at any part of the earth's field.

VI

38. What is the magnetic condition of a bar of soft iron held horizontally above, and parallel to a permanent magnet of the same size resting horizontally on a table?

39. A bar magnet is laid on a table with its north end projecting over the edge. A soft iron ball clings to the under side of the projecting end. State and explain what happens when the south pole of a second magnet is brought above and near to the north pole of the first.

40. You have three equal bar magnets without keepers. How would you arrange them so that, when not in use, they might preserve their magnetism? Give a sketch.

41. If you wish to support a uniformly magnetised bar horizontally on a pivot, how is it that the pivot must be placed nearer to one end than to the other? To which end must it be nearest in this country?

42. A magnet is balanced on a fine point so that it can turn freely in a horizontal plane. When disturbed in its position of equilibrium, it will oscillate from side to side. Upon what does the time depend, taken by the magnet to make one swing?

VII

43. Two equal and equally magnetised bar magnets are fastened together at right angles to each other at their centre—so as to form an equal armed cross. How will the cross set itself when balanced at the middle on a point?

44. A magnet is placed near a compass needle so as to pull the needle a little way round. If a large sheet of soft iron is put between the magnet and the needle, what happens? and why?

45. A bar magnet is laid upon a table, and a soft iron bar of about the same length as the magnet is hung horizontally just above it by a flexible string. What will be the effect on the soft iron bar if a second bar magnet be laid on the table and brought near the first, at right angles to it, and with its north-seeking pole pointing to the middle of the first magnet? Give a sketch explaining the action.

46. How does the position of a "dipping needle" change when it is taken from London (1) towards the north pole, or (2) towards the equator?

VIII

47. A piece of soft iron is brought near to a magnet; how will it be affected? How do you account for the attraction which always takes place between the iron and either pole of the magnet? If a piece of wood or brass be introduced between the magnet and the piece of iron, how will the attraction be affected? Do you know of any material which, if introduced between the iron and the magnet, will destroy or nearly destroy the attraction?

48. What is meant by vertical intensity, horizontal intensity, and total intensity, as applied to terrestrial magnetism? Describe some method of measuring the vertical intensity.

49. Define a unit magnetic pole, a magnetic field of unit strength, the magnetic moment, and intensity of magnetisation of a magnet.

How may the strength of two magnetic fields, each of uniform strength, be compared?

50. Describe some form of magnetometer suitable for determining the horizontal intensity of terrestrial magnetism, and explain how the observations should be conducted.

51. Describe dip and declination.

52. What is meant by a line of magnetic force? Make a sketch showing the arrangement of lines of force *around* and *through* a magnet.

53. Describe the difference between the behaviour of soft iron and that of hard steel when placed in a magnetic field. What is magnetic *permeability?* How is the magnetic character of iron affected by a considerable increase of temperature?

CHAPTER II

ELECTRICITY

SECTION I

IF a plate of ordinary commercial zinc be placed in a vessel containing dilute sulphuric acid, the zinc will be acted upon, and hydrogen bubbles given off. If a plate of pure zinc be placed in the dilute acid no action will take place. It is found that if the ordinary zinc be amalgamated with mercury,—and this can be done by dipping the zinc into acid, and then rubbing it over with mercury,—and then placed in the dilute acid, no action takes place, just as if the zinc were chemically pure. If now a plate of copper be placed in the dilute acid no action will take place. If both the zinc and the copper plates be placed in the dilute acid, but not touching each other, no action is observed. If, however, the two plates be now joined by means of a copper wire, it is found that action takes place, the zinc is attacked, and hydrogen bubbles gradually collect on the copper plate, and eventually bubble up to the surface. It appears from this that the zinc of the zinc plate displaces the hydrogen in the sulphuric acid, forming zinc sulphate; and that the liberated nascent hydrogen dissociates the hydrogen from the adjacent molecule of the hydrogen sulphate, and thus the action is propagated from the zinc to the copper plate, the hydrogen finally being set free at the copper plate.

If now a small magnet suspended by a fine silk fibre be held near to the wire which connects the plates, it will be found to be acted upon as if the wire possessed some magnetic property. Suspending the magnet near to a vertical part of the connecting wire, it will be observed that the magnet tries to set itself at right angles to the line joining its centre to the wire.

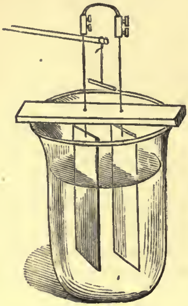


Fig. 35.

The experiment shows that whilst the action is going on, a magnet in the neighbourhood is acted upon by a force—or in other words that whilst the action is going on, a magnetic field is created in its neighbourhood.

After a short time the magnetic effect diminishes, and when this takes place the action in the cell also ceases, and it will be found that the copper plate is covered with hydrogen bubbles. On taking the copper plate out, and thus liberating the bubbles, and replacing the plate, the action, both chemical and magnetic, takes place as before. The cause of the cessation of the action is due to the accumulation of the hydrogen on the copper plate.

Whilst the chemical action takes place, and the magnetic effect is produced, a **current of electricity** is said to circulate around the circuit consisting of the acid, the two plates, and the connecting wire: and it is convenient and usual to say that the action in the cell gives rise to the current of electricity, and the current of electricity gives rise to the magnetic field. As the chemical action begins at the zinc and is transmitted in the fluid towards the copper, the current is said to flow from the zinc towards the copper in the cell, and from the copper to the zinc outside the cell. The exact relation, however, between the observed phenomena—as to which is cause and which is effect, and what is the nature of the mechanism by means of which the phenomena are propagated—is a matter of considerable doubt and is being investigated by modern physicists. In the present chapter we shall not be concerned with the ultimate cause, but only with the relation between the observed phenomena, as experimentally determined. The following chapters, which approach the subject by a different route, and from other experiments, will introduce the student to the seat of the energy and to the mechanism by which the energy is supposed to be propagated.

There are two causes of the diminution of the rate of the chemical action, and hence of the current.

(i) The accumulation of hydrogen on the copper plate. This is called **polarisation.**

(ii) If the zinc plate is impure and is not amalgamated or badly amalgamated, the zinc is attacked locally, and holes formed in it, the hydrogen being set free at the zinc plate. This action is called **local action.** It is due to the presence in the zinc of other metals, chiefly iron. These particles of iron form, with the acid and zinc, small electric circuits, and the action proceeds locally. Local action is avoided either by using quite pure metal, or by thoroughly amalgamating the zinc with mercury.

Before the effects which take place during the passage of a current of electricity can be further studied, it is necessary to find some means for avoiding polarisation.

There are three methods of avoiding polarisation :—

(i) *A mechanical method* of brushing away the bubbles of hydrogen as they are created, or of blowing air across the plate on which they collect. Another mechanical method is to have the plate on which the bubbles collect made with a rough surface, so that as the bubbles are created at the points they more easily detach themselves and rise to the top.

(ii) *A chemical method.*—This consists in placing the plate in a strongly oxidising substance, such as bichromate of potash, or nitric acid.

(iii) *An electrochemical method.*—This consists in placing the copper plate in some solution which will attack the hydrogen and set free copper, and deposit it on the copper plate.

Cells.—Vessels fitted with material to furnish a current of electricity are called **Voltaic or Galvanic cells.**

The more common cells are based in their construction on one or other of the methods given above.

(1) **Smee's Cell** consists of a zinc and platinised silver plate placed in dilute sulphuric acid. The silver being covered with rough platinum the bubbles of hydrogen are set free more readily than from a smooth surface—but the cell is not constant, and the current rapidly falls.

(2) **Bunsen's Cell.**—As this cell will be used very often in the experiments of this chapter, the following is a description of a convenient form of cell.

E is a stone-ware jar $6'' \times 4\frac{3}{8}''$. Placed within this is a sheet of zinc Z, bent into a cylindrical shape, $6\frac{1}{4}'' \times 3''$. To this is fixed a binding-screw. Within this is placed a porous pot P $6\frac{1}{2}'' \times 2\frac{1}{2}''$, and inside this is placed a square rod of compressed carbon C $6\frac{1}{2}'' \times 1\frac{1}{2}'' \times \frac{7}{8}''$. To this is fixed a clamp binding-screw.

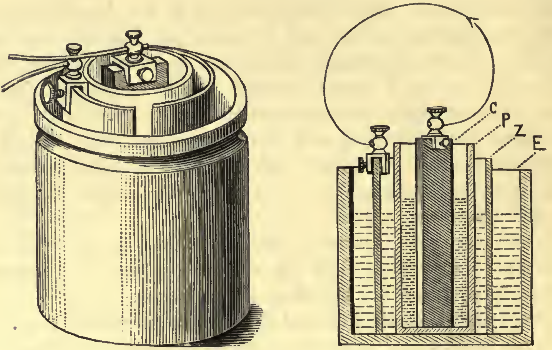


Fig. 36.

The stone-ware jar is filled to within one inch of the top with dilute sulphuric acid: and the porous pot with strong nitric acid.

The two points where the connecting wire is joined to the zinc and carbon are called the **poles** of the cell.

In this cell the zinc displaces the hydrogen in the hydrogen sulphate, and after the porous pot is soaked, the hydrogen decomposes the nitric acid and produces water and peroxide of nitrogen, which latter is partly dissolved in the nitric acid, and partly escapes. This is an objection to the use of the cell, and it is advisable to place the cell in the open air, on a window slab.

Note.—For the successful working of the cell care should be taken to see that all the metal contacts are bright and clean, especially at the zinc and carbon plates. The nitric acid can be used several times, but it gradually grows weak and must be renewed.

(3) **Daniell's Cell.**—This cell will also be much used in this and the succeeding chapters.

The following is the form of the Daniell cell usually met with in laboratories.

C*u* is a cylindrical copper vessel $8''$ deep $\times 3\frac{3}{4}''$ wide, holding about 1 quart. To the copper is soldered a brass binding-screw.

Within the copper vessel is placed a porous pot $8'' \times 2\frac{1}{2}''$, and within this a cylindrical rod of zinc, well amalgamated. To this is connected a connecting screw, with a cross piece of wood, so that the rod can be suspended in the porous pot without touching the bottom of it.

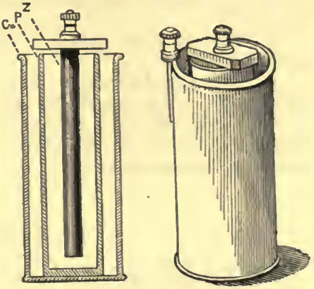


Fig. 37.

The copper vessel is filled with concentrated solution of copper sulphate; and the porous pot with dilute sulphuric acid.

In this cell the zinc displaces the hydrogen from the hydrogen sulphate, and the hydrogen displaces the copper from the copper sulphate, and copper is deposited on the copper vessel. Hence the copper sulphate solution is steadily growing weaker. To avoid this a few crystals of copper sulphate are suspended at the top within the sulphate solution, so that more can be dissolved.

More details as regards these and other cells will be given towards the end of the chapter.

The current from a Daniell and a Bunsen cell will be found approximately constant after they have been in action sufficiently long for the porous pot to become saturated.

EXPT.—*Set up a Bunsen and a Daniell's cell, make a drawing of them and explain their construction.*

EXERCISES

1. What do you understand by a current of electricity? How would you detect its existence? If the wire joining the copper and zinc plates in

the cell were iron, how would you test the existence of a current in the wire?

2. What is the action which takes place in the copper-zinc cell when a current of electricity is passing? If two zinc plates be placed in dilute acid and connected outside by a copper wire, would you expect any action to occur?

3. What is local action? Explain its cause, and show how to prevent it.

4. What is polarisation? State the methods adopted for preventing polarisation, and give examples of each method.

5. Describe Daniell's cell, and state the precautions which must be adopted to get a constant current. How would you tell whether the current were constant or not?

SECTION II

On the Magnetic Field in the Neighbourhood of a Current Circuit

1. On the Field of Force at the Centre of a Plane Circular Circuit.

EXPT. 1.—Take a piece of fairly thick copper wire, and bend it into a circle of about 6″ diameter, and join its ends to the poles of the Bunsen cell, as in Fig. 36.

Place a pivoted compass needle near the centre of the circular coil, and notice the direction towards which the north pole sets when the coil is placed with its plane in the following positions:—

(1) With its plane in the magnetic meridian, and with the carbon pole towards the south, so that the current passes in the wire from the south towards the north. It will be found that the magnetic effect is to set the north pole of the magnet towards the west.

(2) With its plane in the magnetic meridian, but with the carbon pole towards the north. It will be found that the north pole of the magnet sets towards the east.

(3) With its plane perpendicular to the magnetic meridian and with the carbon pole towards the west. The north pole of the magnet will be found to set towards the north (as the earth's force sets the north pole towards the north, the fact that the force due to the current also sets it towards the north can be proved by making and breaking contact at one of the

poles, each time the current is started, the north pole will be found to set more vigorously towards the north).

(4) With its plane perpendicular to the magnetic meridian, but with the carbon pole towards the east. If the force due to the current is greater than that due to the earth, the north pole will be turned to the south. If the force due to the current is less than that due to the earth, the north pole will set towards the north; but by making and breaking contact, it will be seen that the force due to the current tends to set the north pole towards the south.

In each case it will be found that the following rules are true:—

The magnetic force at the centre of a plane coil when a current of electricity is passing through it is

(i) **perpendicular to the plane of the coil;**

(ii) **the sense, or way in which the north pole will set, is given by the following rule:** Imagine a right-handed screw to be screwed through the centre of the coil, perpendicular to the plane of the coil, so that the direction of the current is the direction of rotation or twist of the screw, then the magnetic force at the centre due to the current will tend to set the north pole in the direction of translation of the screw.

To make this rule clearer, suspend a watch in the centre of the coil with its face towards the observer, and so that the current is passing round the wire in the same direction as the direction of rotation of the hands of the watch; and therefore if the screw is screwed through the watch from the front to the back, *i.e.* if the direction of twist is the same as the direction of rotation of the hands of the watch, then the force is such that the north pole is urged from the face of the watch towards the back, *i.e.* in the direction of translation of the screw.

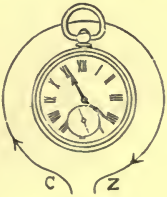


Fig. 38.

This rule may be briefly put thus: **If you twist the screw in the direction of the current, then you screw from south to north of the magnet placed at the**

centre. For if the current is passing in the reverse direction, contrary to that of the hands of the watch, so that the screw would be drawn out from the coil, then the north pole will be urged from the back towards the front, *i.e.* in the direction of translation of the screw.

The experiment is complicated by the action of the earth's force. The effect of the magnetic force due to the current alone can be studied either (i) by making the force due to the current very large compared with that due to the earth's magnetism; or (ii) by so placing a bar magnet that the magnetic force at the centre of the coil, due to the earth and the magnet together, is very small.

EXPT. 2.—(i) Make a coil of only 5 cm. radius, and place the pivoted magnet at its centre, the magnet will now be found to set nearly perpendicularly to the plane of the coil, in whatever position the coil is. Hence test the rule given above for the direction of the current. The same effect can be produced by making a coil of 10 cm. radius and of several turns. Try this and test the law.

(ii) Place a bar magnet in the magnetic meridian, and vertically above the centre of the coil, with its south pole towards the north, and at such a height above the centre, that the centre is near the **zero or neutral point** (see p. 18). When, therefore, the current passes the force at the centre will be almost entirely due to the current. Hence the rule above can be verified.

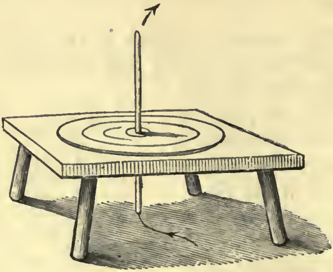


Fig. 39.

2. On the Magnetic Field of Force in the Neighbourhood of a straight Current.

—Take a long straight wire and pass it vertically through the centre of a sheet of glass which is placed on a horizontal stand. Sprinkle some fine iron filings on the glass, and then join the lower terminal of the wire to the carbon pole, and the upper terminal to the zinc pole of a battery of 3 or 4 Bunsen cells. The iron filings will set themselves under the force due to the current, and the lines of force will be found to be circular, with the wire passing through the glass as centre, as in Fig. 40.

The earth's force is not great enough to set the iron filings, and, therefore, if the return wire is far enough off, the force setting the filings may be considered entirely due to the current in the straight circuit. To find out in what direction the north pole sets, take a small magnet and place it near to the wire. If it is placed in the four positions *a*, *b*, *c*, *d*, viz. towards the S, E, N, W respectively of the wire—the current coming up from below—the north pole will be found to set at *a* towards E, at *b* towards N, at *c* towards W, at *d* towards S.

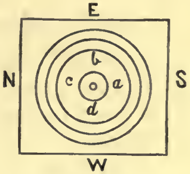


Fig. 40.

If the current be reversed so as to pass downwards, the direction the north pole would set in would be reversed. The following gives the law for the direction of the north pole:—Imagine a right-handed screw to be screwed (so that the screw is passing upwards perpendicular to the plate from below), along the wire in the direction the current is passing, then the north pole of a magnet would be acted upon by a force tending to drive it round in the direction of the rotation of the screw.

Hence the force at a point in the magnetic field near a straight circuit is—1st, perpendicular to the shortest line joining the point to the wire.

2nd. Its sense is related to direction of the current as the direction of rotation of a right-handed screw is to the direction of translation of the screw.

Briefly. Screw a right-handed screw in the direction of the current, and the north pole will be driven round in the direction of the twist of the screw.

That the force at the centre of a plane circular circuit is perpendicular to the plane, and obeys the laws given on page 73, can be shown to follow from the law of the force in the neighbourhood of a straight circuit.

Suppose the plane of the paper to be a plane drawn through the centre of the circular circuit and perpendicular to its plane, so that the current is coming upwards at A, and going downwards at D. Consider the magnetic force at the centre C due to the straight part of the circuit at A. Since the current is coming upwards at A by the rule just given, the force will be in the direction CE (where CE is perpendicular to AB, *i.e.* to the plane of the circuit). Since the current at D is going downwards, by the same rule the force at C is also in the direction of CE.

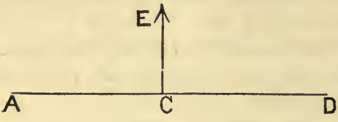


Fig. 41.

Hence the rule given for the direction of the force at the centre of a plane circuit is found to be in harmony with the rule as to the direction of the force in the neighbourhood of a straight circuit.

EXPT. 3.—*To draw the lines of force in a horizontal plane passing through the centre of a circular circuit in which a steady current is passing, the plane of the circuit being in the magnetic meridian.*

Make a circular coil of wire of about 5 turns, and diameter 15 cm., and fix it with its plane vertical on a wooden support. Now, fix upon 3 vertical supports a horizontal board passing through the centre of the coil, and place a sheet of paper on the board. Both the horizontal board and the paper will have to be in two halves, and then placed touching each other. Pass a current of electricity through the wire, and draw the lines of force in the usual way.

3. **On the Magnetic Field in the Neighbourhood of a Spiral Circuit.**—Take some copper wire and twist it round a lead pencil, and thus make a spiral of wire of about 8 inches in length. If the wire is thick enough the pencil can be withdrawn, and the two ends of the spiral bent parallel to the axis and again bent near the centre as in the figure. If the spiral be formed by holding the pencil in one hand, and beginning near that hand twist the wire on in a right-handed screw direction, the spiral will be a left-handed screw; and by twisting it on in the opposite direction a left-handed spiral is formed.

The lines of force in a plane passing through the axis of this spiral can be drawn in the same way as explained in Chap. I. § 2.

The lines of force due to the spiral itself can be shown by means of iron filings. The general nature of these lines is shown in Fig. 42, where it is seen that the lines are parallel within the spiral, and bend out at the ends, so that the field of force created is similar to that due to a bar magnet.

If the end A be joined to the carbon of a Bunsen cell, and the end B to the zinc, and the current at A circulates through the spiral in a right-handed screw direction, then a pivoted magnet placed at A will be found to set with its north pole towards B, so that the spiral appears to behave as a magnet with its south pole at A and north pole at B.

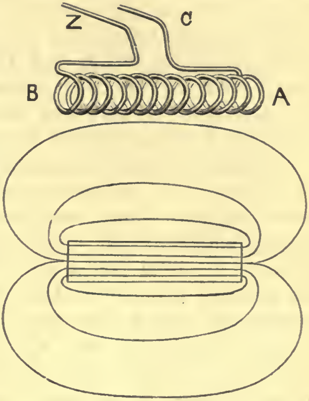


Fig. 42.

By making right and left-handed spirals, and reversing the currents through them it can be shown that

If we screw along the axis of the screw, the direction of the twist being the direction of the current, the direction the north pole will move in will be the direction of translation of the screw, and hence the so-called south and north ends of the spiral can be found.

The intensity of the magnetic field can be considerably increased by placing a rod of soft iron through the spiral. In this way a strong field of force can be created similar to the field of force due to a bar magnet. The spiral with soft iron core is called an **electromagnet**, and a very useful form of it is made by bending a cylindrical bar of iron into a U shape, and wrapping it round with a coil of copper wire.

The study of the action of electromagnets, both as regards the fields of force they create and the action of other fields upon them, forms the branch of electricity called **electro-**

magnetism, and the first principles of it will be given in Chapter IV.

We may give here the following simple actions:—

(1) Take the cell and circular circuit of Expt. 1, and float it on a cork in a large basin of water. The coil will turn so that the lines of force due to it coincide in direction and sense with the earth's lines, *i.e.* it will set with its plane at right angles to the meridian, and in such a manner that if we screw towards the north along the earth's line of force (*i.e.* perpendicular to the circuit) the direction of the twist will be the same as the direction of the current in the circuit.

To follow this more easily we may imagine the coil replaced by a steel disc, magnetised with one face positive and the other negative, and such that if we screw along a line perpendicular to the disc along a line of force, *i.e.* in the direction a north pole would move due to the disc; *i.e.* from the positive face to the negative face, then the direction of the twist must coincide with the direction of the current.

(2) Take the cell and spiral circuit in 3 (p. 77) and float them on a cork in water, the spiral will be found to set with its axis along the meridian, and its "so-called" positive end towards the north.

These experiments illustrate the general principle given in Chapter I. § 2 (p. 22), that the two fields will adjust themselves so that they oppose the least resistance to each other. Hence the coils will set themselves so that the earth's lines of force will coincide as far as possible in direction and sense with the lines of force of the magnetic fields of the current circuit.

In Chapter IV. these relations are more fully investigated. The above experiments were due to M. de la Rive, and the cells may be called de la Rive's cells.

4. **The Galvanoscope.**—We have seen that a magnet placed in a suitable position in the neighbourhood of a wire will detect the presence and direction of a current of electricity.

A coil of wire fixed on a board, with a magnet at its centre free to turn about an axis in the plane of the coil, constitutes an instrument for the detection of the presence and direction of a current and is called a galvanoscope.

EXERCISES

1. A current is passed through a circular coil which is placed in the NE and SW position. The north pole of the magnet at its centre sets towards the NNW, in what direction must the current be passing?

2. How would you arrange a bar magnet so that the north pole in Question 1 will set perpendicular to the coil?

3. If a wire were stretched round the earth at the equator, in what direction (*i.e.* in the same or in the opposite direction to the rotation of the earth) would the current pass in order that a south pole at a place in N latitude would be attracted towards the centre?

4. A wire is placed horizontally in the magnetic meridian, and a current passes from the N towards the S. A small magnet is placed in the same horizontal plane and towards the east of it. In what direction will the magnet tend to set under the influence of the current alone? In what direction will it set if placed vertically below the wire?

5. A current is passing along a telegraph wire, how would you tell, given a small suspended magnet, which way the current was flowing?

6. A current is passing in a wire which runs vertically up the wall of a house, how would you tell whether the lower end was connected to the zinc or carbon of a cell?

7. If the solenoid in page 77 were free to turn about an axis perpendicular to its own axis, which end would set towards the north?

8. A small cell is placed inside a large beaker fitted with a circular coil of several turns. The whole is floated in water. How will it set?

9. If the cell in Question 8 has a solenoid joined to it describe carefully how it will set?

SECTION III

On Measurement of Current. The Ampere.

1. **On the Magnitude of the Magnetic Intensity at the Centre of a Plane Circular Coil through which a Current is passing.**—It will have been noticed that the deflection of a magnet placed at the centre of a coil in which a current is passing is not always the same even for the same coil. For example, it will be noticed that the deflection generally increases from the time the cell is first set up until it becomes fairly constant. If, with the same coil, the deflection increases, the current is said to increase, and in this case, as will be proved in the next section, the zinc in the cell is being consumed at a greater rate.

If the force on the unit magnetic pole at the centre of the coil is found to be doubled, then the current is said to be twice as great; for we are entitled to take any effect of the current as a means of measuring the current. If we measure a current by means of its magnetic effect, it is said to be measured electromagnetically. Two currents are said to be equal if they produce at the centre of the same coil the same magnetic effect, *i.e.* equal forces on the same magnetic pole, and the intensity at the centre of the coil may be taken as a measure of the current.

Now the intensity at the centre can be found by comparing it with the intensity due to a given magnetic field. For the present we will take this field to be that due to the earth—and if we place the coil so that it is in the magnetic meridian, then the intensity at the centre due to the current will be perpendicular to the earth's horizontal intensity, and the magnet will set at an angle to the meridian. Let δ be the angle the magnet makes with the meridian. Let H dynes per unit be the earth's horizontal intensity, and F dynes per unit the intensity due to the current, then we know that $F = H \tan \delta$ (see Chap. I. p. 36).

Hence the intensity at the centre varies as the tangent of the angle of deflection.

We are now in a position to measure current electromagnetically, for we can take as the measure of a current the intensity produced at the centre of a given coil. For example, if the current which produces the unit intensity at the centre of the given coil is taken as the unit of current, then the current which produces at the centre of the same coil i units of intensity, will contain i such units of current.

The following experiment will enable us to pass from one coil to another, *i.e.* enable us to compare currents measured by one coil with currents measured by another, and will suggest a suitable absolute unit of current:—

Arrange six different coils with their planes in the magnetic meridian so that the magnetic intensity due to the current will in each case be perpendicular to the earth's magnetic intensity. Let these coils be A, B, C, D, E, F. Let A, B, C have radii of 17.5 cm, and D, E, F have each half this radius. Let A, D have one turn each; B, E two turns each; and C, F three turns each. Send a current through the six coils. Suppose the deflection of the magnet at the centre of A is 45° (so that the tangent of the deflection is 1), then the deflections of the magnets placed at the centres of the other coils will be given by the following table :—

Coil.	Number of Turns (n).	Radius (r).	Deflection (δ).	Tangent of Deflection ($\tan\delta$).	$\frac{r\tan\delta}{n}$.
A	1	17.5	45	1	17.5
B	2	17.5	63°.20′	2	$\frac{2\times17.5}{2}=17.5$
C	3	17.5	71°.30′	3	$\frac{3\times17.5}{3}=17.5$
D	1	8.75	63°.30′	2	$2\times8.75=17.5$
E	2	8.75	76°	4	$\frac{4\times8.75}{2}=17.5$
F	3	8.75	80°.30′	6	$\frac{6\times8.75}{3}=17.5$

On examining these results we find that the tangent of deflection is proportional to the number of turns in the coil, and inversely proportional to the radius. By means of this relation, therefore, it will be possible to compare the current measured by one coil with the current measured by another. The following example will explain this :—

EX.—Suppose a current measured by a coil of 30 cm. radius and one turn gives a deflection of 45° ; and a current measured by a coil of 20 cm. radius and three turns gives a deflection of 80°.20′, find how many times the second current contains the first.

Since the intensity at the centre of the coil varies as the number of turns, and inversely as the radius, the intensity at the centre of the second coil due to the first current would be $\frac{3\times20\ \mathrm{H}\tan45^\circ}{30}$, *i.e.* 2H; but the intensity due to the second current is H tan 80°.20′, *i.e.* 6H.

$$\therefore\quad \frac{\text{second current}}{\text{first current}}=\frac{6\mathrm{H}}{2\mathrm{H}}=3.$$

It is convenient and necessary to choose a unit of current, and the unit of current adopted is the following :—

That current which, passing through a coil of 1 cm. radius and of one turn, will produce at the centre 2π units of intensity —*i.e.* produce on unit north pole a force of 2π dynes—is called the electromagnetic absolute unit of current.

The reason why 2π appears in the definition is because each arc of the circuit may be supposed to create its own part of the intensity, and since the circumference of the coil is 2π cm., each centimetre of the coil will produce a force of one dyne on the unit north pole.

A current of i units will by this method of measuring currents produce an intensity of $2\pi i$ units. If now this current of i units pass through a coil of n turns and r cm. radius, the intensity at the centre will be $\frac{n2\pi i}{r}$ units, if this intensity is F dynes per unit, then $F=\frac{2\pi ni}{r}$.

It will be seen that the formula $F=\frac{2\pi ni}{r}$ follows from the fact that by the definition of the measure of current the intensity is directly proportional to the current, and by experiment we see that the intensity is directly proportional to the number of turns, and inversely proportional to the radius. The constant 2π is due to the particular unit adopted, *i.e.* if $r=1$ and $n=1$, and the current is the unit of current, then by definition the intensity is 2π units.

2. **To Measure a Current in Absolute Units.**—Place the coil with its plane in the meridian, and measure the deflection δ when the current is passing. Let n be the number of turns in the coil, and r cm. the radius of each turn; F dynes per unit being the intensity due to the current, and H dynes per unit the intensity due to the earth; then

$$F=H\tan\delta,$$

and

$$F=\frac{2\pi ni}{r}.$$

$$\therefore\quad \frac{2\pi ni}{r}=H\tan\delta$$

$$\therefore\quad i=\frac{Hr}{2\pi n}\cdot\tan\delta.$$

Knowing H, and measuring the radius, the value of i can be found from this equation.

A coil of wire fitted with a suspended magnet at the centre, and with means of measuring the deflection, constitutes a **galvanometer.**

The quantity $\frac{2\pi n}{r}$ is a constant for the same coil, and can be determined once for all. It is called the constant of the galvanometer, and is denoted by G. We see that it is the **intensity at the centre due to unit current.**

If the current is i units then $F = Gi$ and $G = \frac{2\pi n}{r}$.

Since
$$F = Gi$$
and
$$F = H \tan \delta$$
$$Gi = H \tan \delta$$
$$i = \frac{H}{G} \tan \delta.$$

If the same galvanometer is used with the same field H, then so long as H does not change, the quantity $\frac{H}{G}$ remains constant, and can be found once for all. It is called the reduction factor of the galvanometer, and is denoted by k.

Hence
$$k = \frac{H}{G}$$
and
$$i = k \tan \delta.$$

The absolute unit of current was for various reasons considered too large, and one-tenth of it has been taken as a practical working unit. This practical unit is called the **Ampere.**

3. Find the intensity at the centre of a coil of 5 turns and 10 cm. radius through which a current of 4 amperes is passing.

Here
$$F = \frac{2\pi n i}{r}$$
and
$$i = \frac{4}{10} \text{ absolute units.}$$
$$\therefore \quad F = \frac{2\pi \times 5 \times 4}{10} = 1.256.$$
$$\therefore \quad \text{The intensity} = 1.256 \text{ dynes per unit.}$$

4. The deflection in a tangent galvanometer of 4 turns and 10 cm. radius is $22\frac{1}{2}°$, and the intensity of the field

at the centre is .18 dynes per unit. Find the current in amperes

$$i=\frac{Hr}{2\pi n}\tan\delta$$
$$=\frac{.18\times10\times10}{2\pi\times4}\tan 22\tfrac{1}{2}^{\circ}.$$

5. **Experiments.**

EXPT. 1.—*Prove the law of the tangent galvanometer, viz. that the intensity at the centre of a circular coil varies as the number of turns and inversely as the radius.*

For this experiment it is convenient to fix on a vertical board four concentric coils, each consisting of a single turn of fairly thick copper wire. Two of these should be about 30 cm. diameter and be placed on opposite sides of the board, the other two having diameters of about 15 cm., and similarly fixed on each side of the board. A semicircular hole should be cut out of the board, so that a compass-box (described on p. 7) can be placed horizontally with the centre of the magnet at the centre of the concentric coils. The terminals of the wires are brought near together, and can be connected as desired by binding-screws. Place the magnetometer so

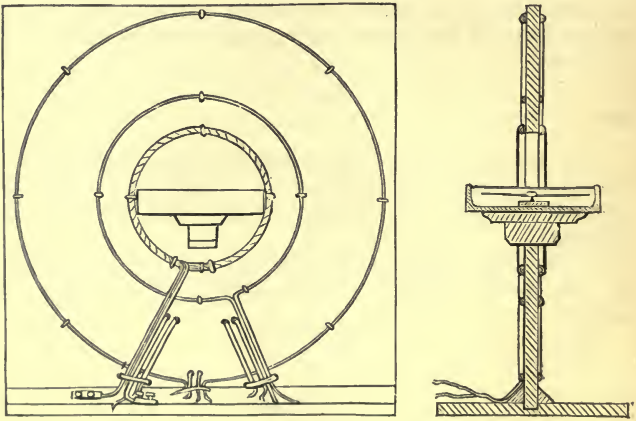


Fig. 43.

that the line on the graduated card is in the plane of the board. Now turn the instrument until the magnet is in the plane of the board, so that the board is in the magnetic meridian, and the brass indicator attached to the magnet will point near the zero of the card. Set up a Bunsen cell and send a current through the two small turns in series, and wait until the deflection is steady. Now read each end of the pointer. Reverse the current at the cell, and take the readings on the opposite side. Take the mean of each pair of readings, and the mean of the two results will be the true deflection. This can be estimated to ¼ a degree. In a similar manner find the deflection when the same current is sent through one short turn; one long turn; two long turns, and enter the results in a table. Assuming $H = .18$, calculate the intensities from the formula $F = H \tan \delta$. Now form another column giving the current.

No. of turns.	Radius.	Readings.			δ.	tan δ.	F = H tan δ.	$i = \frac{Hr}{2\pi n} \tan \delta$.
2	8 cm.	53 53½	54 55	53½ 54¾	54¼	1.389	.2458	.1598
1	8 cm.	35 34½	36 34½	35½ 34½	35	.700	.1259	.1603
2	16.4 cm.	36 35	36½ 35½	36¼ 35¼	35⅝	.716	.1288	.1680
2	16.4 cm.	19½ 20	19 19	19¼ 19½	19⅜	.351	.0631	.1648

An investigation of the results thus tabulated will verify the law.

If the law is true it follows that the current obtained from the relation $i = \frac{Hr}{2\pi n} \tan \delta$ must be the same for each coil. The last column will show that this is very approximately true.

EXPT. 2.—*To find experimentally the current in amperes required to ring an electric bell.*

Set up the galvanometer with its plane in the magnetic meridian. Join the cell up to one of the coils of the galvanometer and to the bell, so as to send a current through the galvanometer and bell. Place in the circuit a piece of thin uncovered German silver wire, and adjust the current by means of the wire until the bell just rings. Now take the readings of the deflection. It may happen that the deflection is too small, in this case use a coil having more turns and of smaller radius. Now measure the diameter of the coils, and find the reduction factor from the relation

$$k = \frac{H}{G} = \frac{Hr}{2\pi n}.$$

Then the current in absolute units is given by

$$i = k \tan \delta$$

and in amperes by

$$i = 10k \tan \delta.$$

6. The Construction of an Electric Bell.

E is an electromagnet fixed to a base. An armature A of soft iron is attached to a spring, so that it will vibrate in front of the poles of the magnet. On the back of the armature is fitted a spring contact maker which makes and breaks contact at B. The current from the cell enters at the binding screw at C*u*, and passes through the contact maker at B to the vibrating tongue, and hence through the coils of the magnet to Z, and hence to the cell. When the current passes the armature A is drawn to the pole and contact at B is broken, and hence the armature is carried back again by the spring. In this way the armature is made to vibrate. To the armature a hammer D is fitted which strikes the bell.

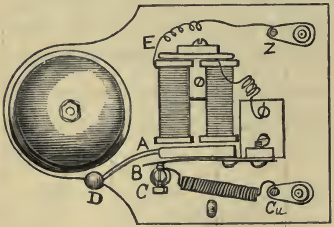


Fig. 44.

7. On the Conditions which have been assumed in proving the Formula for the Tangent Galvanometer.

—(i) The formula $F = \frac{2\pi ni}{r}$ is only true on the assumption that the turns of the coil are all of the same radius, and are all in the same plane. This, of course, cannot be true, but if the mean radius of the coil is large compared with the depth or breadth of the section of the coil, the formula is very approximately true.

(ii) The formula $F = H \tan \delta$ has been proved under two conditions—

(1) The field of force (F) is parallel, so that the intensity at

each end of the magnet for all positions of it is the same as at the centre. This is found to be practically true if the length of the magnet does not exceed one-tenth the diameter of the coil.

(2) In the statical proof of $F = H \tan \delta$ the force due to the friction or torsion of the support of the magnet is neglected. If the couple due to the friction or torsion is denoted by τ, then the condition of equilibrium of the magnet is—

$$FM \cos \delta = HM \sin \delta \pm \tau,$$

$$i.e. \quad F = H \tan \delta \pm \frac{\tau}{M \cos \delta}.$$

In order that the term involving τ should vanish, τ must be small compared with M. Hence the magnetic moment of the magnet should be as great as possible.

8. Recapitulation.

(1) By the definition of the electromagnetic unit of current it follows that—

> The intensity at the centre of a coil of 1 turn and 1 cm. radius when the unit current is passing is 2π dynes per unit.

(2) By the definition of the electromagnetic method of measuring current, viz. that the current is proportional to the intensity at the centre of the coil it follows that—

> The intensity at the centre of a coil of 1 turn and 1 cm. radius when i units of current is passing is $2\pi i$ dynes per unit.

(3) By the experimental fact that the intensity at the centre of a coil due to a given current is *directly* proportional to the *number of turns* of the coil, and *inversely* proportional to the radius, it follows that—

> The intensity at the centre of a coil of n turns and r cm. radius when i units of current is passing is $\frac{2\pi i n}{r}$ dynes per unit

$$\text{or} \qquad F = \frac{2\pi n i}{r}.$$

(4) By the statical relation between the intensities of two perpendicular fields proved in Chapter I. § 4, it follows that—

$$F = H \tan \delta.$$

(5) Hence from (3) and (4) it follows that—

$$i = \frac{Hr}{2\pi n} \tan \delta.$$

(6) The quantity $\frac{Hr}{2\pi n}$ which is constant for same galvanometer and same field is called the **reduction factor** of the galvanometer.

(7) The quantity $\frac{2\pi n}{r}$ which depends only on the galvanometer is called the galvanometer constant, and is denoted by G. Hence

$$F = Gi$$

and

$$G = \frac{2\pi n}{r}.$$

$$i = \frac{H}{G} \tan \delta.$$

(8) $F = \frac{2\pi n i}{r}$ is true under a given condition.

$F = H \tan \delta$ is only true under two conditions.

EXERCISES

1. Describe the construction of an instrument used for measuring currents of electricity by their magnetic effect.

2. A current is passing through the coils of a galvanometer, how would you place the instrument to enable you to compare the intensity of the magnetic field due to the current with the intensity of the earth's field? How would you find the intensity (given $H = .18$)?

3. When is one current said to be double another current? What is a graded galvanometer? A graded galvanometer has three separate coils, and another galvanometer is taken as a standard, such that the unit current is taken to be that current which will give a deflection of 45° in it. Show how you find experimentally what the currents are which produce deflections of 45° in each of the coils of the graded galvanometer.

4. The force at the centre of a coil due to a current in it is proportional to the tangent of the angle the magnet makes with the meridian. What are the conditions which must be satisfied in order that this may be true.

5. If a certain current produces a force of .2 dynes on a unit placed at the centre of a coil of 1 turn and 10 cm. radius, find the force produced at the centre of the following coils by the same current:—

(1) 2 turns of 10 cm. radius.
(2) 4 ,, 10 ,,
(3) 1 ,, 20 ,,
(4) 2 ,, 20 ,,
(5) 4 ,, 20 ,,

Show how to construct an instrument, and how to use it, to verify the law you use in finding these intensities.

6. Why is a tangent galvanometer so called? Show that the deflection of the needle is independent of its magnetic strength.

7. Show that a galvanometer with a single needle may be made more or less sensitive by placing a bar magnet in a suitable position near the instrument.

8. A current is passing through a piece of wire 10 metre long. The wire is formed into a circle, and the intensity at the centre is found to be .05 dynes per unit. If now the wire is formed into a coil of 10 turns, find the intensity at the centre due to the same current.

9. A current is passing through three galvanometers, the first has 1 turn of 10 cm. diameter; second, 3 turns of 20 cm. diameter; third, 6 turns of 9 cm. diameter. The deflection in the first galvanometer is 10°, find the deflection in each of the others. Find also the current in amperes.

10. A current passes through two galvanometers, one is 40 cm. diameter and 2 turns, and the deflection is 10°. The deflection in the other is 20½°; find its reduction factor.

11. A current of 10 units is passing through a coil of 100 turns and radius 20 cm.; find the force at the centre on a north pole of 100 units.

12. A current flowing in a circle of 1 foot radius produces at the centre on m units, a force of 1 grainal (1 ft.-gr.-sec. unit of force); what is the current in C.G.S. units?

13. A current flowing in a circle of L cm. radius produces at the centre on m units of magnetism, a force of F dynes; what is the current?

14. A current is adjusted to give a deflection of 15° in one of the large coils of the galvanometer in Expt. 1, page 84; find the deflection the same current will produce when passing through the two small coils.

15. Find the current in amperes which will give a deflection of 1° in each of the following galvanometers:—

1. 1 turn of 40 cm. radius $H = .18$.
2. 20 turns of 30 cm. radius $H = .18$.
3. 1000 turns of 30 cm. radius $H = .18$.
4. 5000 turns of 5 cm. mean radius $H = .018$.

16. The same current is passed through the following coils: (i) 1 turn of 30 cm. diameter; (ii) 2 turns of 30 cm. diameter; (iii) 3 turns of 30 cm. diameter; (iv) 1 turn of 15 cm. diameter; (v) 2 turns of 15 cm. diameter; (vi) 10 turns of 20 cm. diameter. The deflection produced in No. i. is 5°; find the deflection produced in the others. [9°.55, 14°.41, 9°.55, 19°.16′, 52°.40′.]

17. Show how to prove that the current is the same at all parts of the circuit.

18. A current passing through a circle of 1 cm. radius produces an

intensity of 6.2832 dynes per unit; find the intensity when the turns are 10 and radius 20 cm., and find deflection produced in latter case. [$F=3.1416$; $\delta=88.43^\circ$.]

SECTION IV

Electrochemical Action. Quantity of Electricity

1. **On the Chemical Action which takes place when a Current is passing.**—(*a*) Take a glass or earthenware vessel, and pour into it a dilute solution of copper sulphate (slightly acidulated with sulphuric acid). Take the two copper-wire terminals of a Bunsen cell and dip them into the solution, and observe the action which takes place. It will be found that the copper-wire which is joined to the zinc of the cell will be gradually coated with bright copper, and that the other pole will be gradually dissolved.

(*b*) Now dip the two copper poles into a vessel containing water acidulated with sulphuric acid, and observe the action which takes place. It will be found that gas is given off at both poles. If these gases were collected separately in a manner described later, it would be found that the gas given off at the pole at which the current enters the water—*i.e.* at the pole which is joined to the carbon of the cell—is oxygen; and the gas given off at the other pole is hydrogen. The oxygen as it is given off at the copper pole acts upon the copper, forming oxide of copper—(this is an example of "secondary action"). It is better, therefore, to use platinum wire terminals, when the gases will be given off freely. (It is well to observe here that a single Daniell cell would not decompose the water).

These experiments show that liquids, such as copper-sulphate solution and water, are decomposed by the passage of a current of electricity. Any substance which is decomposed by the passage of a current of electricity is called an **Electrolyte**; and the two components into which it is decomposed are called **ions**; and the action which takes place is called **Electrolysis**. The two poles, which generally consist of metallic wires or plates suspended in the electrolyte, are called **Electrodes**; that at which the current enters the solution is called the **Anode**, and that at which the current leaves the solution is called the **Kathode**. The vessel with the electrolyte and the electrodes constitute an **electrolytic cell**. When a current passes through a solution of copper sulphate, copper is separated from the anode, and an equal mass of copper is displaced from the copper sulphate and deposited on the cathode. When a current passes through water, the water is decomposed into oxygen and hydrogen, the oxygen moving towards the anode, and the hydrogen towards the cathode. Thus oxygen collects at the anode, and hydrogen at the cathode. The ion which is constrained to move in the direction of the current is called electropositive to the other ion. Hydrogen is electropositive to oxygen, and oxygen is electronegative to hydrogen. The action in the case of copper sulphate may be, that the $CuSO_4$ is decomposed into Cu, and SO_4, the copper appearing at the cathode, and the SO_4 at the anode, where it combines with the copper electrode to form $CuSO_4$ (a secondary action)—and therefore copper is electropositive to SO_4 (sulphion).

An electrolytic cell may be used to determine the direction of a current, by observing the nature of the action taking place at the poles. If the cell is arranged so that the mass of the electrolyte, or the mass of an ion, which enters into action in a given time can be measured, it will be shown that the cell can be used to measure the current. An electrolytic cell so arranged is called a **Voltameter**.

The following is a description of three forms of Voltameter :—

(i) *Copper Sulphate Voltameter.*—The electrolyte is a dilute solution of copper sulphate acidulated with sulphuric acid. The electrodes are usually two copper plates, one slightly larger than the other, fitted into a block of wood so as to be parallel to each other. The current passes in at the larger plate so that copper is deposited on the smaller plate. By weighing the smaller plate before and after the current has passed for a known time, the mass of copper deposited can be measured. The size of the plates used depends upon the strength of the current, it being found experimentally that a good deposit is only obtained for a definite relation between the current and the area of the plate. A suitable form for the plate is illustrated in the figure (47). A binding-screw can be attached to the wire; and the tongue can have its upper end fixed into a saw-cut in the block of wood. When in use the whole of the plate should be immersed.

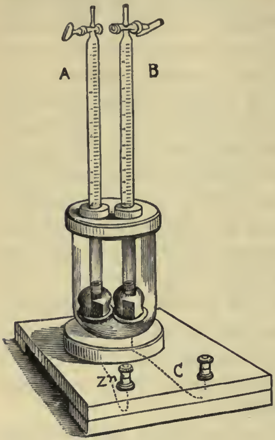


Fig. 45.

(ii) *A Water Voltameter.*—This can be formed by taking a bell-jar, and fitting it with a good cork stopper, and passing through the cork two platinum wires, to the upper ends of which strips of platinum foil are welded. The whole can be rigidly attached to a wooden base, and the two ends of the platinum wires joined to two binding-screws. The gases are collected in two graduated burettes placed vertically, with one of the platinum foils in each, and fixed through two holes in a wooden board.

The vessel is filled with acidulated water, and the water sucked up to fill the burettes completely. After the current has run for a known time, the volume of the gases can be measured, and therefore the masses of each determined.

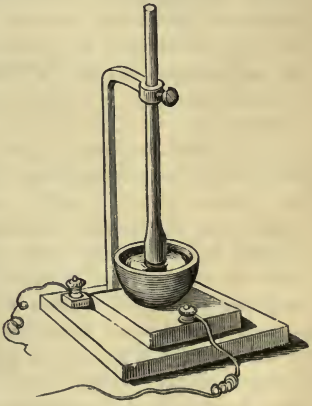


Fig. 46.

(iii) *Silver Nitrate Voltameter.*—This usually consists of a thin platinum cup, which contains the silver nitrate solution. The platinum cup forms the depositing pole, and a silver rod is suspended in the solution to form the other pole. As the current passes silver is deposited on the platinum dish—the mass of which can be measured by weighing the dish before and after the current has been running. The liquid should consist of a neutral solution of pure silver nitrate, containing about 15 parts by weight of salt to 85 parts of water.

2. **The Laws of Electrolysis.**—If a current of one ampere pass through a copper sulphate voltameter for 1000 seconds, it is found that .326 gm. of copper is deposited; if a current of two amperes pass for the same time, it is found that twice the mass of copper is deposited, and more generally it is found that the mass of an ion which enters into chemical action in a given time is directly proportional to the current as measured electromagnetically, *i.e.* a large current is not accompanied by more than a directly proportionally greater deposit. Suppose a current of one ampere to pass for 1000 seconds through each of the following voltameters—a zinc sulphate, zinc chloride, copper sulphate, copper nitrate, water, hydrochloric acid—it will be found that .326 gm. of copper enters into chemical action in each of the copper voltameters; that the equivalent mass $\frac{.326 \times 65}{63.5}$ *i.e.* .336 gm. of zinc enters into action in each of the zinc voltameters; that the equivalent mass of hydrogen .0103 gm. enters into chemical action in both the water and HCl voltameter. This important fact has been carefully investigated by Faraday, and it is established that the same mass of an ion will enter in chemical action, when the same current passes for the same time, whatever the electrolyte may be of which it forms a component. It has also been established by Faraday that the mass of an electrolyte which enters into action during the passage of a current is independent of the form, nature, or size of the electrodes. The nature of the electrodes as regards material and size will effect the *nature* of the deposit—for example, if the current through a copper voltameter is too great, the deposit will be found black and crystalline, and may fall off the plate, but does not effect the mass chemically acted upon. One ampere flowing for one second will set free the same mass of hydrogen from either water or hydrochloric acid, and the same mass of copper from copper sulphate as from copper nitrate. It follows also by the laws of chemical combination that the masses of ions set free by the same

current passing for the same time are proportional to the combining masses of the ions.

These laws were first definitely established by Faraday, and are known as **Faraday's Laws of Electrolysis.** They may be *provisionally* stated as follows :—

I. **The mass of an ion, or of an electrolyte, which enters into chemical action during the passage of a current is directly proportional to the current, and to the time during which it passes, and is independent of the nature or size of the electrodes.**

II. (*a*) **The mass of an ion which enters into chemical action during the passage of a current for a given time is the same whatever the electrolyte may be of which the ion forms a part.**

(*b*) **The masses of electrolytes or of ions which enter into chemical action when a given current passes for a given time are in the ratio of the combining masses of the electrolytes or ions.**

DEFINITION.—The mass of an electrolyte, or of an ion which enters into chemical action during the passage of the unit current for the unit of time, is called the **electrochemical equivalent** of the electrolyte or ion.

The laws of electrolysis state that the electrochemical equivalent (E.C.E.) of an ion is a constant of the ion, and independent of the strength of the current or of the other ions combined with it.

If the current is i units, and time t seconds, and ϵ the E.C.E. of the substance, the first law shows that the mass (M) which enters into chemical action is given by

$$M = \epsilon i t.$$

To verify the laws of Electrolysis experiments should be performed to find the mass of an electrolyte or of an ion which enters into chemical action during the passage of a current with the conditions varied.

I. *With same electrolyte.*

(i) Varying the time and the current.
(ii) Varying nature and size of electrodes.

II. *With different electrolytes.*

The verification of the laws of Electrolysis are complicated by the secondary actions which may take place at the electrodes. When an ion is set free at an electrode it may (if a compound ion) be decomposed, or it may react on the electrodes. Such actions are called **secondary actions**. For example, when SO_4 is set free at the copper plate in the copper sulphate voltameter, it immediately acts upon the copper to form $CuSO_4$.

3. **Quantity of Electricity.**—When a current flows for any length of time in a circuit, a quantity of electricity is said to pass through the circuit.

This language is due to electricity having been at one time regarded as a fluid, and a current of electricity as a flow of this fluid round the circuit. The language is still convenient, and will lead to no danger if no material nature be attributed to the "flow of electricity." It will be shown when we come to static electricity that the modern conception of a current of electricity is a flow of tubes of induction in a medium, the ends of the tubes flowing round the circuit.

When the unit current passes for one second, **the absolute unit quantity of electricity** is said to pass across every section of the wire forming the circuit.

If a current of one ampere pass for one second, then one-tenth of the absolute unit of quantity passes across every section of the circuit in the second. This quantity is the practical unit of quantity, and is called the **Coulomb**.

If the current of i amperes is flowing in a circuit, the quantity of electricity which crosses each section of the circuit in t seconds is ti coulombs. If the quantity is denoted by Q, then

$$Q = ti.$$

The electrochemical equivalent of an electrolyte or of an ion can therefore be defined as the mass of the ion which enters into chemical action when the unit quantity of electricity passes.

The practical unit of E.C.E. is the mass in grammes of the ion which enters into chemical action when a coulomb of electricity has passed, and it is expressed in grammes-per-coulomb.

The E.C.E. of hydrogen is therefore .0000103 gm. per coulomb.
E.C.E. of zinc ,, .000336 ,,
E.C.E. of oxygen ,, $8 \times .0000103$,,

By Faraday's second Law we can find the E.C.E. of any ion or electrolyte given the E.C.E. of hydrogen. The question is the purely chemical one of inquiring what mass of the ion will enter into chemical action when .0000103 gm. of hydrogen enters into chemical action. In other words, if C is the chemical equivalent of the ion compared with hydrogen, then the electrochemical equivalent of the ion is $.0000103 \times C$ gm. per coulomb.

Examples :—

Ex. 1.—*Find the mass of water decomposed by a current of* 2.5 *amperes flowing for* 20 *minutes.*

The E.C.E. of H = .0000103 gm. per coulomb.
E.C.E. of O $= 8 \times .0000103$,,

$\therefore$ Mass of hydrogen set free in 20×60 sec. by 2.5 amperes

$$= .0000103 \times 20 \times 60 \times 2.5 \text{ gm.}$$

Mass of oxygen set free

$$= 8 \times .0000103 \times 20 \times 60 \times 2.5 \text{ gm.}$$

$\therefore$ Mass of water decomposed

$$= 9 \times .0000103 \times 20 \times 60 \times 2.5 \text{ gm.}$$
$$= .2781 \text{ gm.}$$

Ex. 2.—A steady current passes for ten minutes through a copper voltameter, and is found to deposit 20 gm. of copper. Find the strength of the current in amperes.

E.C.E. of copper = .000326 gm. per coulomb.

$$M = i\epsilon t$$
$$\therefore \quad 20 = i \; .000326 \times 10 \times 60$$
$$= i \times .1956.$$
$$i = \frac{20}{.1956} = 102.6 \text{ amperes nearly.}$$

Ex. 3.—The above current is found to deflect a large tangent galvanometer through 4°: find the reduction factor of the galvanometer.

$$i = k \tan d \text{ absolute units.}$$
$$= 10\, k \tan d \text{ amperes.}$$
$$\therefore \quad 10\, k \tan 4^\circ = 200.$$
$$k = \frac{20}{\tan 4^\circ}.$$

Where k is the reduction factor in absolute units.

Ex. 4.—A current of 3 amperes is passed for 30 minutes through acidulated water, find the volume of hydrogen evolved at 750 mm. pressure at 10° C.

The mass of H evolved

$$= .0000103 \times 3 \times 30 \times 60 \text{ gm.}$$

And the vol. of this at 750 mm. and 10° C.

$$= \frac{283 \times 760 \times .0000103 \times 3 \times 30 \times 60 \times 1000}{273 \times 750 \times .0896} \text{ cc.}$$
$$= 652 \text{ cc.}$$

Expt. 1.—*To find the E.C.E. of copper.*

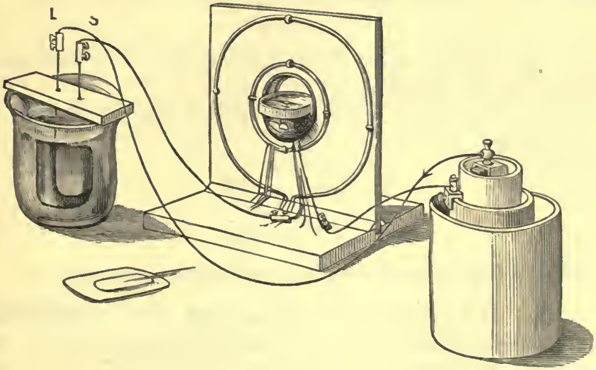


Fig. 47.

Set up a copper voltameter, and a tangent galvanometer, and send a current through the galvanometer and the voltameter; *i.e.*, arrange that the current passes through the galvanometer and the voltameter, so that the current which passes through the voltameter can be measured by the galvanometer. Make a preliminary experiment for a few minutes to see (i) if the current remains fairly constant—*i.e.*, if the deflection of the galvanometer remains constant; (ii) if the deposit is taking place satisfactorily—*i.e.*, clear firm deposit. If this is the case, take out the plate upon which the deposit is formed, and carefully dry it and weigh it. Now replace it, and let the current run for about thirty minutes, and read the deflection of the galvanometer every minute. At the end of the time, which must be carefully observed, take out the plate, dry it, and weigh it.

Calculation

(1) *To find the current.*—To find the average deflection of the galvanometer, add all the deflections together, and divide by the number. This will give the average deflection which may be used, if the current does not vary more than a few degrees.

Measure the radius of the coils, and find the reduction factor in amperes.

If δ is the average deflection, and k the reduction factor in amperes, then the current i is given by

$$i = k \tan \delta.$$

(ii) *To find the mass deposited.*—This is the difference between the two weighings of the plate.

Let the mass deposited be m grammes, and let the time of depositing be t sec.

then since $m = \epsilon i t$

$$\therefore \quad m = \epsilon t k \tan \delta$$

$$\therefore \quad \epsilon = \frac{m}{tk \tan \delta}.$$

This will give the E.C.E. in grammes per coulomb.

4. On the Use of Voltameters for measuring Current.—A voltameter can be used to measure directly the quantity of electricity which passes through a circuit in a given time. All that is necessary is to introduce the voltameter (*e.g.* a copper voltameter) into the circuit, and measure the mass of the ion set free or deposited.

Suppose the copper voltameter is used, and the mass deposited is found to be m grammes, then

$$Q = \frac{m}{\epsilon} \text{ where } \epsilon = \text{E.C.E. of the ion (copper).}$$

Again, if i amperes is the average current, then

$$Q = it$$
$$\therefore \quad i = \frac{m}{\epsilon \times t}.$$

If therefore t is measured, this will give the average current; and if the current is constant during the time, this is a satisfactory method of measuring the current, instead of using a galvanometer.

The *two advantages of a voltameter* as compared with a galvanometer are—

(i) No determination of the strength of the magnetic field is necessary.

(ii) There is no difficulty in measuring the angle of deflection.

But the disadvantage is that it does not indicate any changes in the current, but measures only the average current.

EXPT. 2.—*To find the reduction factor of a tangent galvanometer by means of a voltameter.*

Set up the tangent galvanometer and voltameter in circuit with a Bunsen cell as in Expt. 1, and find as in that experiment the mass of copper deposited in a given time, and read the angle of deflection; then from the equations

$$m = i\epsilon t$$
$$i = k \tan \delta$$

we get

$$k = \frac{i}{\tan \delta} = \frac{m}{\epsilon t \tan \delta}.$$

If ϵ is taken in gm. per coulomb, then k will be found in amperes.

5. On the Chemical Action in a Galvanic Cell.—When a current passes through a Daniell, a Bunsen, or any galvanic cell, chemical action takes place, and this chemical action obeys the laws of electrolysis. When one ampere passes through a Daniell cell for 1000 seconds, .336 gm. of zinc is dissolved, and .0103 gm. of hydrogen is displaced from the sulphuric acid, which in its turn displaces .326 gm. of copper from the copper sulphate, and .326 gm. of copper is deposited on the copper plate.

In the Bunsen's cell, when one ampere passes for 1000 seconds, .336 gm. of zinc is consumed, and .0103 gm. of hydrogen is set free to act upon the nitric acid, forming water and nitric peroxide gas, which is partly dissolved by the nitric acid, and partly given off in fumes.

It is important to note that for every cell using zinc, the mass of zinc consumed per coulomb is always the same, viz., ·000336 gm.

Expt. 3.—*To find the mass of zinc dissolved, and the mass of copper deposited during the passage of a current of electricity for a given time, through a Daniell cell.*

Construct a Daniell cell, having a thin zinc rod placed in sulphuric acid contained in a porous pot. Place the porous pot in an earthenware vessel containing saturated copper sulphate solution, and, bending a thin sheet of copper into a cylinder, place it in the copper sulphate.

Weigh the zinc rod, and the copper plate before the current passes, and let the current pass through a tangent galvanometer for about thirty minutes, and read the deflection.

Now weigh the zinc rod and copper plate. Find in the case of the copper the mass of copper deposited per coulomb. This will be found to be .000326 gm.

The mass of zinc consumed per coulomb ought to be found .000336 gm., but the surface of the zinc rod becomes granular, and particles of zinc are apt to drop off, and a good result cannot be obtained.

Exercises

1. Describe the fundamental facts of electrolysis.

2. What is an electrolyte, and what is an electrolytic cell? Describe the copper sulphate and silver nitrate electrolytic cell, and explain the chemical actions which take place during electrolysis.

3. Give a sketch of the apparatus required for the electrolysis of water provided with means for measuring the products of electrolysis. What is the fundamental difference in the nature of the action which goes on during the electrolysis of water and of copper sulphate respectively?

4. You are provided with copper sulphate, copper plate, and wire, and a tangent galvanometer,—describe the experiments which must be performed to show that the mass of sulphate electrolysed in a given time (say half an hour) is independent of the size of the plates, of their relative

position, and of the strength of the solution. Also describe experiments to prove that the mass electrolysed is directly proportional to the current.

State Faraday's First Law of Electrolysis, and show that an electrolytic cell may be used to measure a current.

5. A copper sulphate voltameter and a water voltameter are arranged in series, and the same current passes through them for the same time, what is the relation between the masses of the electrolyte decomposed, and between the masses of the ions separated? If 1 gm. of water is electrolysed, how many gms. of sulphate are electrolysed, and how many gms. of copper deposited and of oxygen and hydrogen set free?

6. What is Faraday's Second Law of Electrolysis? A copper sulphate and a silver nitrate voltameter are used to measure currents. Show that Faraday's Second Law enables us to compare the two currents. Ex.—

$$\left.\begin{array}{l}\text{2 gm. of Copper deposited in 20 minutes.}\\ \text{2 gm. of Silver deposited in 30 minutes.}\end{array}\right\}\text{Compare the currents.}$$

7. Describe the experiments which must be performed to prove Law II.

8. Find the E.C.E. of the following ions, given that the E.C.E. of hydrogen is .000103 gm. per unit quantity of electricity :—

Iron, Zn, Cl, S, C, SO_4, HCl, H_2O, HNO_3, NO_3. . .

9. A current of 3 amperes is passed through a solution of $CuSO_4$, find the mass of copper deposited in one hour.

10. A current of 100 amperes is passed through a $CuSO_4$ voltameter, find the mass of $CuSO_4$ decomposed in 2 minutes.

11. A current of electricity is passed through a $ZnSO_4$ voltameter for 3 minutes and decomposes 2 gm., find the current in amperes. What mass of water would be decomposed by the same current in the same time?

12. A current of 10 amperes is passed for 20 minutes through an electrolyte, and decomposes 4 gm. of it, find the E.C.E. of the electrolyte.

13. A current is passed through a copper sulphate voltameter and through a tangent galvanometer of two turns (radius = 12 cm.), the deflection being 20° ; find the mass of copper deposited in 30 minutes.

14. What current must be employed to generate 100 cc. of hydrogen at 750 mm. pressure, and 15° C. in 20 minutes?

15. What current must be passed through water to decompose 8 gm. of it in 5 hours?

16. Find the reduction factor of a tangent galvanometer, given that a current which deposits .43 gm. of copper in 30 minutes gives a deflection of 20°.

17. A current of 3 amperes is given by a Daniell's cell, find the mass of zinc used in 2 hours, and the mass of copper sulphate which must be supplied in order to keep the copper sulphate solution concentrated.

18. Describe the action of a Daniell's cell.

SECTION V

Work done in the Circuit and by the Cell. Electromotive Force.

1. **Electromotive Force. Voltage.**—The student will have observed that whilst the current is passing, heat is generated in the circuit, the temperature of the wire and of the cell rising. The quantity of heat generated in a part of the circuit is easily measured, by placing the part in a given mass of water, and noticing the rise of temperature in a given time. Suppose the current is 2 amperes, and that 150 gm. of water has its temperature raised from 12° C. to 16° C. in 30 minutes. In this case the heat created in the part of the circuit immersed in the water is 150×4 (w.gm.C.°) units, *i.e.* 600 units. Now Joule has determined that to raise the temperature of 1 lb. of water through one Centigrade degree requires an expenditure of 1390 ft.-lbs.-wt. of energy, hence to raise the temperature of 1 gm. of water through 1 C.°, *i.e.* to create 1 (w.gm.C.°) unit of heat, requires an expenditure of 4.2×10^7 (*i.e.* about 42,000,000) ergs. Again, during the 30 minutes, $2 \times 30 \times 60$ coulombs of electricity is said to pass along the circuit. Hence the work done in the part of the circuit immersed in the water is $\frac{4.2 \times 600 \times 10^7}{2 \times 30 \times 60}$ ergs for every coulomb of electricity which passes, *i.e.* $.7 \times 10^7$ erg per coulomb.

It is found convenient to take as a practical unit of energy 10^7 (*i.e.* 10,000,000) ergs; this practical unit is called a **Joule.** It is shown in the Appendix that a joule is equal to .737 ft.-lbs.-wt.

Heat is molecular energy, and therefore it can be measured in ft.-lbs.-wt., in ergs, or in joules. An erg of heat is an excessively small unit; it would only raise the temperature of a gramme of water through a forty-millionth part of a Centigrade degree. A joule of heat would raise the temperature of a gramme of water through $\frac{1}{4.2}$ C.°, *i.e.* roughly $\frac{1}{4}$ C°.

In the example above, the energy given out in the part of the circuit is .7 joule per coulomb.

DEFINITION.—**The work done in a circuit, or between two points of a circuit, when unit quantity of electricity passes is called the Electromotive Force of the circuit, or the electromotive force between the two points of the circuit.**

Electromotive force is abbreviated thus—E.M.F.

When the work is expressed in ergs, and the quantity of electricity in absolute units, the E.M.F. is given in absolute units. The absolute unit of E.M.F. is the erg-per-unit quantity of electricity.

When the work done is expressed in joules, and the quantity of electricity in coulombs, the E.M.F. is expressed in joules-per-coulomb. This practical unit of E.M.F. is called a **Volt.**

The E.M.F. is a volt when a joule of work is done for every coulomb. Hence the E.M.F. of the part of the circuit in the example taken is .7 joule-per-coulomb, *i.e.* .7 volt.

A volt is 10^7 ergs per $\frac{1}{10}$ absolute units, *i.e.* 10^8 ergs per absolute unit, *i.e.* 10^8 absolute units of E.M.F.

As the E.M.F. of a circuit or part of a circuit is practically expressed in volts, the term **Voltage** is often conveniently used to denote the E.M.F. of the circuit or part of the circuit.

EXPT. 1.—*To find the E.M.F. between two points of a circuit.*

In order to find the E.M.F. between two points of a circuit, it is necessary—

(i) To measure the energy in joules given out in a known time in the part of the circuit. This can be done by immersing the part of the circuit in a calorimeter containing water, and measuring the heat created in w.gm.C.°, and hence in ergs and joules.

(ii) To measure the coulombs of electricity which pass in the time. This can be done by letting the current pass through a tangent galvanometer, and measuring the current, and the time during which it passes, or it can be done by allowing the current to pass through a $CuSO_4$ voltameter.

Set up a Bunsen cell, and short circuit it for a brief time.

Weigh out into a calorimeter about 150 gm. of water, and let it remain until it has taken up the temperature of the room.

Now take a piece of German silver wire (silk covered) of 18 B.W. gauge, and between 30 to 70 cm. long. Solder its two ends to thick copper wires; wind the wire into an open spiral, and pass the copper wires through a piece of wood. Place the G. S. wire immersed in the water, and connect its terminals to the Bunsen cell, through a tangent galvanometer (using such turns as will give a suitable deflection). Take the time and temperature, and allow the current to pass for about thirty minutes. During the time the current is passing it may change; it is advisable therefore to take the temperature and deflection of the galvanometer every two minutes and find the average deflection.

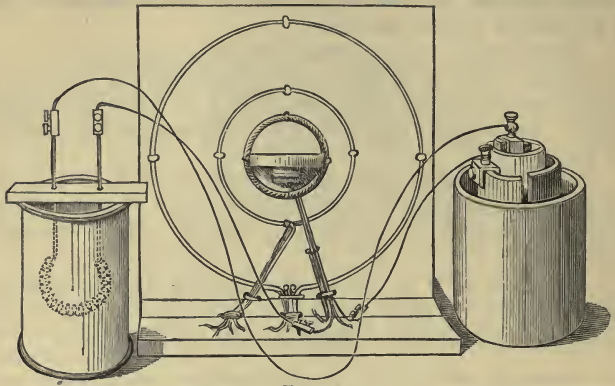


Fig. 48.

Example:—

Mass of water . . . . = 150 gm.
Mass of calorimeter . . . = 40 gm.
Time during which the current is passing . = 30 minutes.
Temperature at beginning . . . = 15° C.
Temperature at end . . . = 20° C.
Average deflection . . . . = 55°.

To perform the calculation :—

(1) Find the heat generated in the water.

If the specific heat of the copper calorimeter be taken as .1, the water equivalent of the calorimeter is 4 gm. Hence the heat absorbed by the water and calorimeter is

$$\begin{aligned} &154 \times (20-15) \text{ w.gm.C.}^\circ \text{ units.} \\ &= 770 \text{ (w.gm.C.}^\circ\text{) units.} \\ &= 770 \times 4.2 \text{ joules.} \end{aligned}$$

(2) To find the coulombs which have passed.

Find the reduction factor of the galvanometer in amperes by measuring the radius (r) of its coil, observing the number (n) of the turns, and taking

$$H = .18$$

and using the formula

$$k = \frac{10.Hr}{2\pi n}.$$

Then find the current from the formula

$$i = k \tan \delta.$$

Suppose

$$\begin{aligned} &r = 10 \text{ cm.} \\ &n = 2 \\ &k = \frac{10 \times .18 \times 10}{2 . \pi . 2} = \frac{10 \times .18 \times 10}{2 \times 3.1416 \times 2} = 2.86 \\ &i = k \tan 55^\circ = 2.86 \times 1.428 = 4.08 \text{ amperes.} \end{aligned}$$

Hence the number of coulombs which pass in thirty minutes is 4.08 × 30 × 60.

Hence 770 × 4.2 joules of work is done in the circuit when 4.08 × 30 × 60 coulombs pass; this is at the rate of $\frac{770 \times 4.2}{4.08 \times 30 \times 60}$, *i.e.* .44 joule per coulomb.

Hence the E.M.F. of this part of the circuit *when this particular current* is passing is .44 volt.

If the E.M.F. is E units, and the quantity of electricity which passes is Q units, the work done is EQ units, or W = EQ.

(α) If E and Q are measured in absolute units, then the work done is E.Q ergs.

(β) If E is measured in volts, and Q in coulombs, the work is E.Q joules, *i.e.* .737 × E.Q ft.-lbs.-wt.

If the current is i units, and time t seconds, $Q = it$ and $W = Eit$.

The work done per second is the **power** absorbed by the circuit.

The absolute unit of power is the erg-per-second. The practical unit of power used in electrical investigations is the joule-per-second, *i.e.* 10^7 ergs-per-second. This practical unit of power is called a **Watt.** If the E.M.F. is a volt, and the current is an ampere, then the quantity of electricity passing in one second is a coulomb, and therefore the work done per second is a joule. Hence a watt may be called a volt-ampere.

Since $W = Eit$ ∴ power $= Ei$, and therefore E.M.F. is the power absorbed in the part of a circuit per unit current; in practical units it is the watt per ampere.

The practical unit of power used in general engineering is the horse-power, *i.e.* 33,000 ft.-lbs.-wt. per minute, or 550 ft.-lbs.-wt. per second.

$$\begin{aligned} \text{Now a watt} &= \text{a joule per sec.} \\ &= 10^7 \text{ ergs per sec.} \\ &= .737 \text{ ft.-lbs.-wt. per sec.} \end{aligned}$$

$$\therefore \quad 1 \text{ ft.-lb.-wt. per sec.} = \frac{1}{\cdot 737} \text{ watts} = .135 \text{ watt,}$$

$$\therefore \quad 1 \text{ horse-power} = \frac{550}{\cdot 737} \text{ watts} = 746 \text{ watts.}$$

EXERCISE 1.—The E.M.F. of an electric lamp is 100 volts when a current of .64 amperes is passing through it; find the power absorbed by it.

$$\text{Work done in 1 sec.} = 100 \times .64 \text{ joules}$$

$$\therefore \quad \text{Power} = 64 \text{ watts}$$

$$\therefore \quad \text{H.-P.} = \frac{64}{746}.$$

Hence the horse-power required for forty such lamps $= \frac{40 \times 64}{746} = 3 \cdot 43.$

EXERCISE 2.—A current of 2 amperes is sent through a wire which is placed in 200 gm. of water, and the temperature is raised from 10° C. to 40° C. in twenty minutes. What is the voltage?

$$\text{Heat created} = 200\,(40 - 10)\ \text{w.gm.C.}^\circ\ \text{units}$$
$$= 6000\ \text{w.gm.C.}^\circ\ \text{units}$$
$$= 6000 \times 4.2\ \text{joules}$$
$$\therefore\quad \text{Work done per sec.} = \frac{6000 \times 4.2}{20 \times 60}\ \text{joules,}$$
$$= 21\ \text{joules.}$$
$$\therefore\quad \text{In 1 sec. 2 coulombs pass.}$$
$$\therefore\quad \text{Work done per coulomb} = \frac{21}{2} = 10.5\ \text{joules.}$$
$$\therefore\quad \text{The voltage} = 10.5\ \text{joules per coulomb.}$$
$$= 10.5\ \text{volts.}$$

2. **The Electromotive Force of Cells.**—Two questions now arise: (1) Where does the energy come from which is found given out in the circuit when a current is passing? Where is the agent supplying this energy, which we have seen given out in the form of heat? and (2) what is the means by which this agent transmits the energy to the circuit? Where is the mechanism transmitting the energy?

Two analogies will perhaps make these questions clearer. Energy is found given off at the lathes of a workshop—the parts of the metals are separated from each other, and work is done; in this case also, the greater proportion of the energy is transformed into heat, and the turnings are found very hot. In this case we know that the energy is supplied by burning coals in a boiler, the energy is derived from a storage of what is called chemical energy. This energy is first of all transformed into heat at a high temperature in the fire-box; and part of this is transmitted into the water in the boiler and into the steam created, and by means of this steam the energy (at least part of it) is transmitted to the piston of a steam-engine, and hence by pulleys and belts to the lathe. In this case the coal and the air in which it burns is the agent, and the boiler, steam-engine, and belting constitute the mechanism which transmits the energy to the lathes. Only a very small fraction of the total energy given out by the coals is transmitted to the lathes. There is a considerable loss in the transmission. Heat escapes up the chimney and in all directions, so that there is a vast loss between the fire-box and the water: again there is a great loss in transmission by the steam, heat being radiated into space: and lastly there is a loss by transmission by the other mechanisms, by what is called friction of the machinery, necessitating a plentiful supply of oil. Another point to be noticed which will be of use afterwards is this: the transmission of the energy takes place from the hot parts towards the cold, from the hot fire to the steam in the boiler, then to the cylinder. The piston also moves from the hot part of the cylinder to the cold. From this point of view the transmission of energy may be said to be due to the difference of temperature between the parts.

In the case of a cell the supply of energy is traced to the chemical energy of its constituents. The supply of energy is due to the action of the sulphuric acid on the zinc, or to the combustion of the zinc. In the case of the Daniell cell energy is given out by the combustion of zinc in sulphuric acid, and a less amount is absorbed in separating the copper from copper-sulphate and depositing it on the copper plate; and possibly some other quantity becomes stored up (as potential energy) in the cell. The difference between the quantity evolved and that absorbed is the energy left to be transmitted, and it is this energy which is found evolved in the form of heat at the different parts of the circuit. In a Bunsen cell energy is evolved by the combustion of zinc in sulphuric acid, and energy is absorbed in the complicated action which takes place at the carbon plate, and the difference is that which is found given out in the circuit. When the unit quantity of electricity passes through the circuit of a Daniell's it is known that .000336 gm. of zinc is consumed, and that when the same quantity of electricity passes through a Bunsen's cell the same mass of zinc is consumed. But the combustion of a given mass of zinc in the process which takes place in the more active Bunsen cell produces nearly twice as much energy as is produced by the combustion of an equal mass of zinc by the process which takes place in a Daniell's cell. Hence when the unit quantity of electricity passes through a Bunsen's cell, about twice the quantity of work is done as when the unit quantity passes through a Daniell's cell.

We may compare with this the difference there is between the quantity of heat which may be obtained from boilers of different types (different both as regards the number and shape of tubes, the nature of the draught, etc.) by the combustion of the same mass of coal. Looked at from this point of view the electromotive force of a cell is analagous to the *duty* of a boiler and engine—*i.e.* the work done by the boiler and engine per lb. of coal—generally expressed in horse-power-hour per lb. of coal, or in lbs. of coal per horse-power-hour, and this is the measure of the economical value of the agent. We express the E.M.F. of a cell in terms of the joules of work done per coulomb, *i.e.* in terms of the work done per a given unit mass of zinc consumed. In the Bunsen cell the work absorbed in the nitric acid and at the carbon plate is less than the work absorbed in separating the copper from the copper sulphate and depositing it upon the copper plate of the Daniell cell. Hence the energy left over per gramme of zinc consumed in the Bunsen cell is greater than in the Daniell cell.

The student must carefully distinguish between the electromotive force of a cell or of an electrical agent and the energy of the cell, or agent.

The electromotive force is measured by the work done per coulomb (*i.e.* per definite mass of zinc consumed), whilst the energy of the cell is the total work the cell is capable of doing until all the zinc is consumed, or until the action ceases on account of some other cause.

The electromotive force of a Daniell cell will always be the same whatever its size may be—pint, quart, or gallon, but the energy of the cells will be different.

DEFINITION.—**The quantity of work which is done in the circuit of a cell when the unit quantity of electricity passes round it, is called the Electromotive force of the cell.**

Dr. Joule has found that when 65 gm. of zinc is consumed in a Daniell's cell, 47670 w.gm.C.° units of heat are evolved. Hence the quantity of heat evolved when .00336 gm. of zinc enters into action is $.000336 \times \frac{47670}{65}$ w.gm.C.° units, *i.e.*

$$.000336 \times 733 \times 4.2 \times 10^6 \text{ ergs, } i.e. \ 1.039 \times 10^8 \text{ ergs.}$$

Hence the work done when the unit quantity of electricity passes through a Daniell's cell is 1.039×10^8 ergs.

$$\therefore \text{ E.M.F. of Daniell's} = 1.039 \times 10^8 \text{ ergs per absolute unit.}$$
$$= 1.039 \text{ volts.}$$

Hence the E.M.F. of a Daniell cell is approximately a volt, and in a similar way it can be proved that the E.M.F. of a Bunsen cell is approximately 2 volts.

This is worked out on the assumption that all the energy derived from the chemical action appears in the form of what we may call electrical energy, *i.e.* the energy which may be said to be due to the current. That is, we assume, that none of the energy due to the chemical action becomes latent as potential energy, or sets free any latent energy which may have been in the cell. This is very nearly true in the case of a Daniell's cell, where the constituents of the cell remain in the same condition during the passage of a current.

3. **The Energy and E.M.F. required to decompose a given Mass of Water.**—When water is decomposed it is separated into hydrogen and oxygen. Now Favre and Silberman found that when 2 gm. of hydrogen combined with 16 gm. of oxygen, the heat developed was 68920 (w.gm.C.°) units. Now the chemical equivalent mass of zinc is 65 gm., and when 65 gm. of zinc are consumed in a Daniell cell the heat set free is 47670 w.gm.C.° units. If now a Daniell cell be connected in series with a water voltameter, and we imagine it possible for a current to pass, so that hydrogen and oxygen are set free—the energy supplied by the combustion of 65 gm. of zinc would be 47670 w.gm.C.° units of heat, but the energy required for the decomposition of the equivalent mass of water would be 68920 w.gm.C.° units of heat. Hence one Daniell cell cannot decompose water. In fact it will be seen that the least E.M.F. possible for the decomposition of water is $\frac{68920}{47670}$, *i.e.* 1.44 times the E.M.F. of the Daniell cell.

The mechanism by which the energy is conveyed from the cell, and absorbed in the circuit is not so easy to identify. Modern investigations point to the ether surrounding the cell as the means by which the energy is transmitted. Whilst the combustion is taking place in the cell, waves are set up in the ether by the molecules of the cell, and the energy is transmitted by these waves. Now, as in the case of the heat-waves, only such bodies become heated which are capable of absorbing the waves, so the energy of the cell only appears where a body is in a condition for absorbing these waves; and it is supposed that the complete circuit is such a body.

In the present chapter we shall only be concerned with the relation between the E.M.F., the current, and the nature of the conducting circuit. These relations will be established by experiment, and we shall not be concerned with the ultimate cause of the observed facts.

The E.M.F. of a cell need not necessarily remain constant during the time a current is running. The action which takes place in a cell may be such as to impede and ultimately stop the action. For example, in the simple cell of § 1, page 68, there is a continual accumulation of hydrogen taking place, and hence the conditions are continually changing; instead of copper being presented to the acid, hydrogen is presented. In this case the E.M.F., *i.e.* the rate work is done per coulomb, rapidly diminishes towards a very small quantity, in much the same way that a fire becomes choked with cinders, and combustion becomes slow. The same is the case with the Leclanche cell, owing to the accumulation of hydrogen in the oxide of manganese. The Daniell cell and the Bunsen cell (especially the former) remain during the action in almost the same condition—there is little or no polarisation. Hence the E. M. F.'s of these cells are very nearly constant whilst supplying current.

4. The following remarks may be useful as giving a practical idea of the quantities introduced in this chapter :—

(i.) A ft.-lb.-wt. is the work done in lifting 1 lb. a height of 1 foot. An erg is the work done in overcoming 1 dyne through 1 cm., or roughly in lifting a milligramme 1 cm.

A joule is approximately the work done in lifting $\frac{3}{4}$ lb. a height of 1 foot.

4.2 joules is 3.095 ft.-lbs.-wt., *i.e.* approximately the work done in lifting 3 lbs. a height of 1 foot ; and this is the work required to raise the temperature of only 1 gramme of water 1 C.°

To raise the temperature of a gallon (10 lbs.) of water from 10° C. to 100° C. requires $1390 \times 10 \times 90$ ft.-lbs.-wt. of work, *i.e.* 1,668,000 joules of work or 1,258,000 ft.-lbs.-wt. Rankine, in his book on the *Steam Engine*, Chapter II. Part I., tells us that a man turning a winch for eight hours can do 1,296,000 ft.-lbs.-wt. of work, *i.e.* a man, if set to churn water, could just raise 1 gallon of water from 10° C. to boiling point in eight hours—neglecting the loss by radiation during the time.

Now 1,258,000 ft.-lbs.-wt. is 1,668,000 joules. And when .000,336 gm. of zinc is consumed in a Daniell cell, 10^8 ergs, *i.e.* 10 joules of work is done. Hence when 33.6 gm. of zinc is consumed 10^6 (a million) joules of work is done. Hence when 65 gm. of zinc is consumed 1,668,000 joules of work is done. In other words, the consumption of 65 gm. of zinc in a Daniell cell creates as much heat as would boil the gallon of water (again assuming no loss of heat by cooling). Assuming that only half the heat created by the zinc can be used for the purpose, then 112 gm. of zinc would have to be used, or rather more than $\frac{1}{4}$ lb. The zinc rod of a quart Daniell weighs about 3 lbs. Hence a quart Daniell would boil about 12 gallons of water, or do as much work in this way as twelve men working eight hours. It may be remarked that the 3 lbs. of zinc would cost 1s., the twelve men for eight hours about 36s., and the coal for the purpose probably a fraction of a penny.

(ii.) Power is the rate of doing work. The practical standard of power is the horse-power. The horse-power is 33,000 ft.-lbs.-wt. per minute, or 550 ft.-lbs.-wt. per second. If an agent work as hard as a standard horse, he would have to be able to lift (say) 55 lbs. a height of 10 feet every second. Now from Rankine (*Steam Engine*, Chapter III. Part I.), we find that a draught-horse drawing a cart can do 432 ft.-lbs.-wt. per second, and that a man turning a crank can only do 45 ft.-lbs.-wt. per second—*i.e.* a good draught horse can work at $\frac{9}{11}$ horse-power, and a good man can work at a rate of $\frac{1}{10}$ of the draught-horse, or about $\frac{1}{10}$ horse-power. If the man does his work by lifting weights he can only do half as much work in the same time as if he turns a crank, *i.e.* he can do about 22 ft.-lbs.-wt. per second. The student would do well to gain a practical idea of power by lifting up and down a 28 lb. weight a height of a foot twice in a second continuously for a few minutes—he would then appreciate the difference between work and power, and would understand that in order to keep a lamp burning bright work must be done at a *definite* rate.

The watt—the electrical unit of power—is a joule per second, *i.e.* $\frac{3}{4}$ ft.-lbs.-wt. per second. In order to keep a 16 candle-power incandescent lamp burning brightly heat must be supplied to the carbon filament at the rate of 64 watts, *i.e.* 48 ft.-lbs.-wt. per second. We have seen that a man turning a crank can just do this. We will see later that the greatest rate a Quart Daniell can supply heat to the outside circuit is about $\frac{1}{2}$ joule per second; and that the greatest rate a Quart Bunsen can supply it is 5 joules per second.

EXERCISES

1. What is meant by a (w.lb.C.°) unit of heat; and by (w.gm.C.°) unit? How many (w.gm.C.°) units are there in a w.lb.C.° unit?

2. Describe an experiment to prove that it requires an expenditure of 1390 ft.-lbs.-wt. of energy to raise the temperature of 1 lb. of water 1 C.°

3. What is a dyne? an erg? Find the number of dynes in a lb.-wt., and the number of ergs in a ft.-lb.-wt.

How many ergs of work is required to raise the temperature of 1 gm. of water, 1 C.° given that 1390 ft.-lbs.-wt. will raise 1 lb. of water 1 C.°?

4. What is a joule? How many joules are required to create a w.gm.C.° unit of heat? Find the energy in joules required to make the following changes—

(*a*) 10 kilogm. of water from 0° C. to 100° C.
(*b*) 10 kilogm. of water at 0° C. to steam at 100° C.
(*c*) 100 lbs. of water at 15° C. to 30° C.

5. Prove that 1 joule = .737 ft.-lbs.-wt.

6. A current of 4 amperes in 10 minutes raises the temperature of 200 gm. of water from 15° C. to 30° C.; find (i) the joules of work done; (ii) the coulombs of electricity which pass; (iii) the E.M.F. or voltage of the circuit.

7. Define electromotive force between two points of a conductor. What is the volt? What is the absolute unit of E.M.F.? A wire has a current of electricity passing through it, show how to find its E.M.F.

8. Define agent, energy of an agent, power of an agent? What is a machine? What is meant by transmission of energy? Give examples of various methods adopted for transmitting energy, showing where loss of energy takes place in the transmission.

9. What is the measure of the economical value of an agent, *e.g.* a steam-engine or a water-turbine?

10. What is the source of the energy found given out, in the form of heat, in the circuit of a cell? How much energy does a Daniell cell supply when 10 gm. of zinc is consumed?

11. If zinc is 4d. per lb., what is the least cost of boiling a gallon of water from 15° C. to 100° C.; using a Daniell cell, and supposing all the heat to be given to the water?

12. If zinc is 4d. per lb., and Daniell cells are used to supply the energy of $\frac{1}{6}$ horse-power for one hour, find the zinc used and the cost of it.

13. Explain why, when the unit quantity of electricity passes through a circuit, the quantity of work done is not the same for different cells.

14. What is a watt? An incandescent lamp requires a current of .64 ampere, and its E.M.F. is 100 volts. How many watts does it require? What horse-power does it require; and what weight falling 105 feet in the second would do the work?

SECTION VI

On the Relation between the E.M.F. in a Circuit, and the Current flowing through the Circuit. Ohm's Law.

1. **Ohm's Law.**—With the apparatus which has been used in Expt. 1, § 5, the E.M.F. between the ends of a given piece of wire, can be determined when currents of different magnitude pass through it. The current can be diminished by adding wire to the circuit, and it may be increased by adding cells to the circuit. In each case the heat created in the same piece of wire can be found: and the E.M.F. in volts determined. A table can now be formed giving the current, and the corresponding E.M.F.'s. The following is such a table:—

Current.	E.M.F.	$\frac{\text{E.M.F.}}{\text{Current.}}$
Amperes.	Volts.	
1	.25	.25
2	.5	.25
3	.75	.25
4	.1	.25

It will always be found that the ratio of the E.M.F. to the current is constant for the same conductor; *i.e.* the third column in the above table will be constant. In other words, the E.M.F. between the terminals of a given conductor, when a current is passing, is directly proportional to the current. This relation was discovered by Ohm in 1827, by an altogether different method, and is called Ohm's Law.

Ohm's Law.—The E.M.F. between the terminals of a given conductor in which there is a steady current, is directly proportional to the current passing through the conductor, provided that the conductor remains in the same physical condition.

If R is the ratio of the E.M.F. (E) in volts to the current (*i*) in amperes which is passing through a given conductor; this experiment shows that R is constant.

$$\therefore \quad \frac{E}{i} = R$$

or

$$E = iR.$$

$\frac{V}{I} =$ [illegible]

This is the mathematical expression of Ohm's Law. The quantity R may be determined once for all for a given conductor; and it is a constant of the conductor, depending upon some property of the conductor. This property of the conductor is called its resistance, from the fact that the greater it is the less the current is for the same E.M.F.

The practical unit of resistance is the resistance of a conductor in which, when one ampere flows, the E.M.F. is 1 volt. This practical unit is called the **Ohm.**

If a current of 1 ampere is passing through a conductor, the number of volts of E.M.F. between its terminals, is the number of ohms resistance. In the example given above, the E.M.F. is .75 volt for a current of three amperes, *i.e.* at the rate of .25 volt per ampere; ∴ the resistance is .25 ohm.

The absolute C.G.S. unit of resistance is the resistance of a conductor in which when the absolute unit of current is passing the E.M.F. is the absolute unit of E.M.F.; viz. one erg per absolute unit of quantity.

To find the number of absolute units of resistance in the ohm; *i.e.* to find the resistance of a conductor which is such that when a current of 1 ampere (*i.e.* $\frac{1}{10}$ absolute unit) is passing the E.M.F. is 1 volt (*i.e.* 10^8 absolute units).

From the equation $\frac{E}{i} = R$

$$R = \frac{10^8}{10^{-1}} = 10^9$$

∴ the resistance of the ohm is 10^9 absolute units. The ohm is therefore 1000 million absolute units of resistance.

EXPT. 1.—*To verify Ohm's Law, and find the resistance of a given piece of G.S. wire in ohms.*

Set up the apparatus used in Expt. 1, page 104, and find the E.M.F. between the terminals of the wire when currents of approximately 1 ampere, 2 amperes, 3 amperes pass through the wire. Make a table as on p. 113, giving the current, E.M.F., and ratio of E.M.F. to the current. This ratio will be found approximately constant, and, as seen above, it is the resistance of the wire in ohms.

Note.—This is a direct and absolute method of verifying Ohm's Law, and of finding the resistance of a conductor in absolute measure. It is not, however, capable of great accuracy, and indirect methods of testing the accuracy of Ohm's Law are employed.

2. **Joule's Law.**—The experiment which has been performed in order to verify Ohm's Law was originally performed by Joule in order to prove that the heat created in a conductor is directly proportional to the square of the current, and to the time during which the current passes. This law can therefore be deduced from Ohm's Law, and the results of the experiment above can be arranged so as to verify Joule's Law.

In determining the E.M.F. in the experiment from the heat created we make use of the following relation (where J is Joule's dynamical equivalent of heat, *i.e.* 4.2 joules per w.gm.C.° unit of heat).

$$Eit = JH$$

and hence

$$E = \frac{JH}{it}$$

But the experiment shows that $\frac{E}{i}$ is constant and equal R.

Now

$$\frac{E}{i} = \frac{JH}{i^2 t}$$

$$\therefore \quad \frac{JH}{i^2 t} \text{ is constant and equals R}$$

$$\therefore \quad JH = Ri^2 t.$$

i.e. for the same conductor we have the following :—

Joule's Law.—The heat generated in a given conductor is directly proportional to the square of the current, and to the time the current passes, and to the resistance of the conductor.

If the current is in amperes, the resistance in ohms, and the time in seconds, the heat created is i^2Rt joules.

If the current is in absolute units, the resistance in absolute units, and the time in seconds, the heat created is i^2Rt ergs.

Now it takes 4.2 joules to create 1 w.gm.C.° unit of heat. Hence the heat created is

$$\frac{i^2Rt}{4.2} \text{ w.gm.C.° unit,}$$

i.e.

$$.24\, i^2Rt \text{ w.gm.C.° unit.}$$

In a resistance of R ohms through which a current i amperes is passing, heat is created at the rate of i^2R joules per sec.; *i.e.* the power given out in the conductor is i^2R watts. To maintain this, energy must be supplied or work done at the rate of i^2R watts. From this point of view the resistance of a conductor may be measured by the power required to maintain a current of 1 ampere in it—this power being required on account of the property the conductor possesses of absorbing energy when a current is passing through it, and this property may be defined as the resistance of the conductor. The practical unit of resistance would be that resistance which required a power of 1 watt to *maintain* a current of 1 ampere; *i.e.* a power of 1 joule per sec. to maintain 1 ampere; *i.e.* a conductor on which 1 joule of work is done when 1 ampere passes for one second; *i.e.* a conductor on which when 1 ampere passes the E.M.F. is 1 volt; and this is the practical unit already adopted and called the ohm.

3. Arc and Incandescent Lamps.

If a current be sent through a short piece of fine platinum wire, heat will be created in it, and the temperature of the wire will rise until the rate of loss of heat by radiation is equal to the rate of the gain of heat. If the resistance is R ohms, and the current i amperes, then the temperature of the wire will be constant when the rate of loss of heat is equal to i^2R joules per sec. If the mass of the wire is small, and if the wire is a good radiator of heat, the wire can easily be made brilliantly luminous. This is the principle of the **incandescent lamp**, which consists of a fine filament or wire of carbon, hermetically sealed within an exhausted bulb. A lamp which will give a light equal to 16 standard candles, *i.e.* a lamp of 16 candle-power, requires heat to be supplied to it at the rate of 50 watts. Now, this power may be supplied by a small current through a larger resistance, when the E.M.F. will be large. It is usual to make lamps of the same candle-power, but of different voltage, usually 50 or 100 volts. A 50 volt 16 candle-power lamp will require a current of 1 ampere in order that the power may be 50 watts, and the resistance of such a lamp will be 50 ohms. A 100 volt 16 candle-power lamp will require only a current of half an ampere, and the resistance will be 200 ohms, and therefore a longer and finer filament will be required.

The **arc light** is the same in principle as the incandescent light; but in this case the air and the particles of carbon between two carbon poles are made luminous by the passage of a great current of electricity, about 10 amperes, and to produce a great candle-power the resistance must be great. Since, however, all the particles between the poles are made luminous, and since the rate of loss of heat is much less than in the case of the incandescent filament, it requires considerably less power per candle-power in the case of the arc light than in the case of the incandescent. An arc lamp requiring 10 amperes, and a voltage of 50 volts will give 1000 candle-power, *i.e.* 500 watts for 1000 candles, *i.e.* 2 candles per watt instead of nearly 4 as in the case of the incandescent lamp.

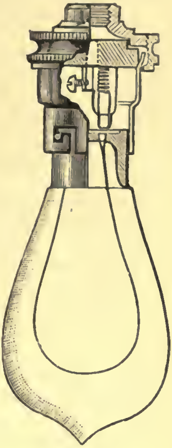


Fig. 49.

EXPT. 2.—*To verify Joule's Law.*

This experiment is the same as that to verify Ohm's Law. Find the heat created (in joules) between the ends of the conductor when currents approximately 1, 2, 3 amperes pass through it for a given time, and make a table showing the current, the heat created in the given time, and the ratio of the heat created to the square of the current. This ratio will be found constant.

EX. 1.—A conductor of 2 ohms resistance is placed in 150 gm. of water, and a current of 3 amperes passed through it for 30 minutes: find the heat created and the consequent rise of temperature.

Heat created $= i^2Rt$ joules.

$= 3^2 \times 2 \times 30 \times 60$ joules.

And this creates $\frac{3^2 \times 2 \times 30 \times 60}{4.2}$ w.gm.C.°

If x C.° is the increase of temperature produced by this in the 150 gm.,

$$150 \times x = \frac{9 \times 2 \times 30 \times 60}{4.2}$$

$$x = \frac{9 \times 2 \times 30 \times 60}{4.2 \times 150} = 52,$$

$\therefore$ the rise of temperature = 52 C.°

Ex. 2.—Find the power required to maintain a current of .6 ampere in a resistance of 100 ohms.

Work done in t sec. $= i^2Rt$ joules

$= (.6)^2 . 100 . t$ joules.

$\therefore$ Power $= .36 \times 100$ watts

$= 36$ watts.

Ex. 3.—Find H.P. required to feed 40 50-candle-power lamps, each requiring 100 volts and 2 amperes.

Power required for each lamp

$= Ei$ watts

$= 100 \times 2$ watts

$= 200$ watts.

Hence power required $= 40 \times 200$ watts

$= \frac{40 \times 200}{746}$ H.P. = 10.7 H.P.

Ex. 4.—A current is measured by finding the rise in temperature in 400 gm. of water due to this current passing through a resistance of 2 ohms immersed in the water. If the rise is from 10° C. to 30° C. in ten minutes, find the current.

Heat created $= 400 \times (30 - 10)$ w.gm.C.° units

$= 400 \times 20 \times 4.2$ joules.

Let current be i amperes. Then since

$$JH = i^2Rt$$

$$400 \times 20 \times 4.2 = i^2 . 2 \times 10 \times 60.$$

$$i^2 = \frac{400 \times 20 \times 4.2}{2 \times 10 \times 60}$$

$$= 28.$$

$$i = \sqrt{28} = 5.3 \text{ amperes.}$$

Ex. 5.—A current of electricity passes through a $CuSO_4$ voltameter for $1\frac{1}{2}$ hours, and also through a wire of 2 ohms resistance, which is immersed in 1000 gm. of water. Find the rise of temperature when .88 gm. of copper has been deposited.

E.C.E. of copper $= .000326$ gm. per coulomb. Hence when .88 gm. of copper is deposited $\frac{.88}{.000326}$ coulombs must pass, and since this passes in 90×60 sec., the current must be $\frac{.88}{.000326 \times 90 \times 60}$ amperes; *i.e.* .5 ampere (nearly).

But

$$JH = i^2Rt.$$

$$\therefore \quad H = \frac{i^2Rt}{J}$$

$$= \frac{(5.)^2 \times 2 \times 60 \times 90}{4.2} \text{ w.gm.C.° units}$$

$$= 642.85 \text{ w.gm.C.°}$$

And this raises 1000 gm. of water through

$$\frac{642.85}{1000} \text{ C.°}$$

i.e. $\quad .642$ C.°

Ex. 6.—A current of 3 amperes passes for half an hour through a resistance of 2 ohms, find the rise in temperature of a mass of 2000 gm. of water in which the wire is immersed.

$$JH = i^2Rt$$

$$= 3^2 \times 2 \times 30 \times 60 \text{ joules.}$$

$$H = \frac{3^2 \times 2 \times 30 \times 60}{4.2} \text{ w.gm.C.°}$$

$$= \frac{54000}{7} \text{ w.gm.C.°}$$

$\therefore$ rise of temperature, due to this heat (assuming all the heat to be employed in raising the temperature of the water)

is

$$\frac{54000}{7 \times 2000} \text{ C.°}$$

$$= 3.8 \text{ C.°}$$

4. The following remarks may perhaps be useful, as giving a practical idea of the various quantities occurring in this chapter:—

(i) A ft.-lb.-wt. is the work done when a pound is lifted one foot. If this is done once a second, then the work is done at the rate of 1 ft.-lb.-wt. per sec., *i.e.* the power of the agent is 1 ft.-lb.-wt. per sec.

Hence, when an agent performs work, in order to find the work done by it there are three quantities which must be known. These are (1) the force the agent can exert; (2) the speed at which the agent can exert its force; (3) the time during which the agent acts. If the agent can exert a force of P lbs.-wt., at the rate of V feet per sec., for t sec., then the work done is PVt ft.-lbs.-wt.; and the power is PV ft.-lbs.-wt. per sec. Agents differ not only as to their power, but also as to the force they are capable of exerting. Two steam engines may have the same power, but one may be capable of exerting a great force at a slow rate, the other a small force at a great rate. Again, two men may have the same power for performing mechanical work, but one of them may be capable of exerting a force of 40 lbs.-wt. at the rate of 1 ft. per sec.; the other may be capable of exerting a force of 20 lbs.-wt., but at the rate of 2 ft. per sec. If the second man were set to lift a load of 160 lbs.-wt., he would have to employ a machine by which a small force exerted at a great rate would be transformed into a larger force exerted at a slower rate; for example, he might use a system of pulley-blocks of velocity ratio 8, by means of which a force of 20 lbs.-wt. at 2 ft. per sec. would be transformed into a force of 160 lbs.-wt. at $\frac{2}{8}$, *i.e.* $\frac{1}{4}$ ft. per sec. (neglecting loss by friction and other causes).

The following are the chief characteristics of an agent:—

(1) The power of the agent. This of course varies within certain limits with the work which has to be done—*i.e.* with the load—*i.e.* the resistance to be overcome.

(2) The quality or nature of the work done—*i.e.* the magnitude of the force and the speed.

(3) The efficiency, *i.e.* the ratio of the work done to the energy supplied to the agent.

(4) The total work which can be done by the agent; this of course depends upon the supply of energy; in the case of a steam engine it depends on the supply of coal; in the case of a cell it depends upon the supply of zinc.

(5) In the case of a living agent, the power and work done depends upon the time during which the work goes on.

If an agent is to be employed to overcome a resistance greater than that which it can do *directly*, it must be made to transmit its energy through a machine. The province of a machine is to transmit and transform the energy of an agent. The following properties of a machine must be known :—

(1) The ratio in which it changes the velocity at which the resistance is overcome—this is called the velocity ratio of the machine.

(2) The ratio in which it changes the magnitude of the resistance,—this is called its mechanical advantage.

(3) The efficiency of the machine.

A galvanic cell is an agent capable of doing work, and the conducting wire, with the ether surrounding it, may be provisionally regarded as the mechanism by which the energy of the cell is distributed.

The power of the agent may be expressed in terms of the E.M.F. and the current—*i.e.* Ei watts—corresponding to the expression PV ft.-lbs.-wt. per sec.

5. Recapitulation.

Notation.

i = Current in amperes.
Q = Quantity of electricity in coulombs.
E = E.M.F. or voltage in volts.
W = Work or energy in joules.
H = Heat in w.gm.C.° units.
R = Resistance in ohms.
Π = Power in watts.
J = Joules equivalent, *i.e.* 4.2 Joules per w.gm.C.°

The following relations depend upon—

I. The definition of quantity of electricity. Hence **Q = it.**

II. Definition of E.M.F. as the work done per coulomb. Hence **W = EQ.**

III. The relation between the heat created and the work required to create it. Hence **W = JH.**

IV. The relation between the E.M.F. and current of a given conductor, *i.e.* Ohm's Law **E = iR.**

V. Algebraic deductions.

No. Equation.	How Obtained.	Equations.	Unit.	Quantity.
1	Def. I	$Q=it$	Coulomb	Electricity
2	From 2	$i=\frac{Q}{t}$	Ampere	Current
3	Def. II	$W=EQ$	Joules	Work
4	Experiment III.	$W=JH$	Joule	Work
5	From 1 and 3	$W=Eit$	Joule	Work
6	From 4 and 5	$JH=Eit$	Joule	Work
Ohm's Law				
7	Experiment IV.	$\frac{E}{i}=R$	Ohm	Resistance
8	From 7	$E=iR$	Volt	E.M.F.
9	From 7	$i=\frac{E}{R}$	Ampere	Current
10	From 6 and 7 or direct experiment and 9	$W=JH=i^2Rt$	Joule	Work
		$W=JH=\frac{E^2t}{R}$	Joule	Work
Power				
12	Def.	$\Pi=\frac{W}{t}$	Watt	Power
13	From 12	$W=\Pi t$	Joule	Work
14	From 5 and 12	$\Pi=Ei$	Volt-Ampere or Watt	Power
	From 4 and 8	$\Pi=i^2R$	Watt	Power
		$\Pi=\frac{E^2}{R}$	Watt	Power

EXERCISES

1. What is the relation between the voltage (E.M.F.) of a given piece of wire and the current passing through it? How could this relation be established by experiment? Point out the sources of error in the experiment.

2. State Ohm's Law, and state what you understand by resistance.

3. Define the "ohm," and show how to find the resistance of a given wire in ohms.

4. State Joule's Law, and show how to verify it. Show how to deduce Joule's Law from Ohm's Law.

5. Find E.M.F. (or voltage) of a conductor of 120 ohms through which .6 amperes is flowing.

6. A conductor has an E.M.F. of 100 volts, and a current of 2.5 amperes, find its resistance.

7. A 100 volt lamp absorbs 64 watts, find its resistance.

8. A current of 2 amperes passes through a wire immersed in 200 gm. of water, and the temperature rises 6° C. in 30 min., find the E.M.F. in volts, and resistance of the wire in ohms.

9. If a current of 4 amperes were passed through the wire in question 4, what would the new E.M.F. be; and what would be the rise of temperature in the same mass of water in the same time?

10. A wire of 10 ohms resistance is immersed in 100,000 gm. of water, and a current of 15 amperes is passed through it; find the rise of temperature in 10 minutes.

11. A resistance of 50 ohms has a current of .64 ampere passing through it, and the temperature remains constant. At what rate is it radiating heat.

12. A current of 10 amperes is sent through a piece of platinum wire of .5 ohm resistance. Find the rate at which heat is radiated from it when the temperature is constant.

13. A current of 12 amperes passes through a wire and raises the temperature of 150 gm. of water 5.5 C.° in 4 minutes. Find the resistance of the wire.

14. A Bunsen cell drives a current of 3 amperes through a wire of .5 ohm resistance. If the E.M.F. of the cell is 2 volts, compare the work done in the wire to the work supplied by the cell in the same time.

SECTION VII

On the Practical Standards of Resistance, Current, and E.M.F., and the direct Application of Ohm's Law to a Simple Circuit.

1. **Practical standards.** (i) **Resistance.**—The method which has been given above of determining the resistance of a conductor is liable to many errors, and several other methods have been adopted which are capable of greater accuracy. It would be tedious, and not always possible, to measure every resistance by the absolute method; much the same as it would be to always refer to astronomical measurements in order to find the time of day. The practical method is to compare the resistance of any given conductor with the resistance of some conductor — the resistance of which is known, and methods of doing this will be given in the following sections of this chapter. We must therefore adopt some conductor of convenient form to serve as a standard of resistance. The resistance of this conductor may either be determined absolutely, or the resistance of some other conductor of more

convenient form for the purpose may be thus determined, and the resistance of the standard measured by comparison with it. Before the question of electrical standards was taken up by the British Association experimenters and manufacturers adopted units of their own. Lenz, for example, adopted as his unit a piece of copper wire 1 foot long of No. 11. Jacobi adopted 25 feet of a certain copper wire, weighing 345 grains; and he sent to Poggendorff and others copies of this unit, expressing his wish that they might express all their results in terms of it. This was the beginning of standards, and this standard is known as Jacobi's Standard. "The first effect of the commercial use of resistance was to turn the 'feet' of the laboratory into 'miles' of telegraph wire. Thus we find employed as units, in England the mile of No. 16 copper wire, in Germany the German mile of No. 8 iron wire, in France the kilometre of iron wire of 4 millimetre diameter." To facilitate comparison boxes of coils were formed which contained various multiples of the unit adopted. None of these units could be reproduced because it is now known that the resistance of substances varies with their physical condition, and varies with the temperature: and the objection to using any of them as a standard is that the resistance of copper and iron changes with time. To remedy these defects Dr. Werner Siemens in 1860 employed as standard a column of chemically pure mercury one metre long and one square millimetre section. This is known as the Siemens's unit. In 1861 the British Association appointed a committee to determine the best standard of electrical resistance. The units which had been hitherto adopted were arbitrary, depending upon the size and mass of some more or less suitable material. Previous to 1861 Weber had worked out the system of absolute units which has been explained in the previous sections, and the British Association committee determined to adopt as the unit of resistance the absolute ohm, as defined in Section VI., and to construct practical standards of resistance which should have a resistance as near as could be measured equal to the unit ohm. These standards were constructed of platinum, gold-silver alloy, platinum-iridium alloy, and in mercury. The solid wires are wires from .5 to .8 mm. diameter, and the mercury standard consists of a glass tube about $\frac{3}{4}$ metre in length.

This unit is known as the British Association unit. Considerable advance has been made in the accuracy of the methods for measuring resistance in absolute measure, and the International Commission on Electrical Units, assembled at Paris in 1884, adopted as a *practical* unit, under the name of the *legal* ohm, the resistance of a column of mercury of one square millimetre in cross-section and 106 cm. in length at temperature of 0° C. In 1891 the British Board of Trade adopted a legal standard, which takes its place amongst the legal standards of length and mass. The following are the definitions :—

(1) The standard of electrical resistance should be denominated the ohm, and should have the value of 1,000,000,000, in terms of the centimetre and second.

(2) That the resistance offered to an unvarying electric current by a column of mercury of a constant cross-sectional area of one square millimetre, and of a length of 106.3 centimetres at the temperature of melting ice may be adopted as one ohm.

(3) That the value of the standard of resistance constructed by a committee of the British Association for the Advancement of Science in the years 1863 and 1864, and known as the British Association unit, may be taken as .9866 of the ohm.

(4) That a material standard, constructed in solid metal, and verified by comparison with the British Association unit, should be adopted as the standard ohm.

The following are the various more or less arbitrary units which were adopted before the introduction of the B.A. units—given in terms of the B.A. unit :—

Jacobi, .6367 B.A. unit; Siemens, .9537 B.A. unit; French kilometre of iron wire, 9.760 B.A. units; English copper mile, 13.59 B.A. units; German mile, 57.44 B.A. units.

Copies of the ohm (legal or B.A. ohm) and of multiples and submultiples of it are made of some wire well covered with double silk covering. The wire used is German silver or platenoid, substances which are found to change little in resistance with change of temperature. The wire is doubled and then wound on to a bobbin, and well covered with paraffin wax. The double winding is to prevent the bobbin becoming an electromagnet when a current passes through it, or to prevent induction of electric currents.

For convenience of use a set of bobbins containing the ohm, and multiples of the ohm, are fixed in a box, and the arrangement for joining them up is shown in the figure.

One end of the wire on the first bobbin is soldered to the brass piece A, the other end being soldered to the brass piece B; the one end of the wire on the second bobbin is soldered to the brass B, the other end to the brass C, and so on. These pieces of brass can be well connected by brass plugs. When all the plugs are in place making connection between the brasses, there is practically no resistance between the first and last piece of brass of the series.

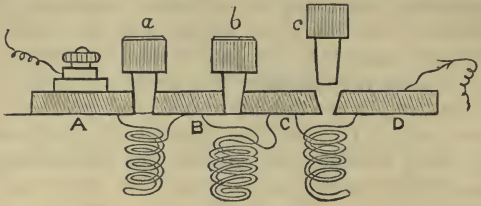


Fig. 50

If any plug be taken out then the current must pass through the wire between the two adjacent pieces of brass. If a box is furnished with bobbins having respectively 1, 2, 3, 4, 10, 20, 30, 40, 100, 200, 300, 400 ohms wound on them, it will be seen that any resistance between 1 and 1110 can be obtained.

A simpler form of resistance box can be easily made by taking a large bobbin, and winding on it 1, 2, 3, 4, 10, 20, 30, 40 ohms, and connecting up the wires to binding screws, instead of the pieces of brass as explained above, and making the necessary connections by short pieces of thick copper wire.

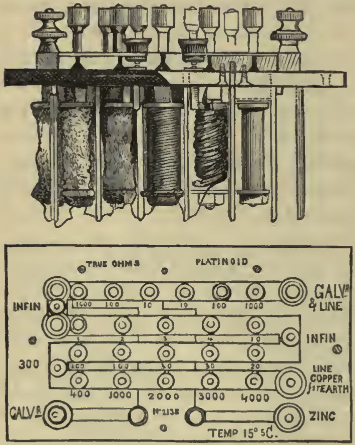


Fig. 51.

(ii) **Current.** — The measurement of current based on its magnetic effect is not found to be one which admits of a suitable standard being chosen.

The difficulty arises from many sources—

(1) It is difficult to measure the value of H at the centre of the coil.

(2) The coil must be large, and the intensity at the centre will be small, and this introduces errors arising from the torsion or friction of the suspension.

The standard of current is, therefore, derived from its electrolytic effect. We might define the practical unit quantity of electricity to be that quantity which would deposit a unit mass of copper in a copper-sulphate voltameter, or a unit mass of silver in a silver-nitrate voltameter.

The Board of Trade Standard of Current is given in the following orders :—

(1) That the standard of electrical current should be denominated the ampere, and should have the value of one-tenth (0.1), in terms of the centimetre, gramme, second.

(2) That an unvarying current which, when passed through a solution of nitrate of silver in water, in accordance with the specification attached, deposits silver at the rate of 0.001118 of a gramme per second, may be taken as a current of one ampere.

(iii) **Electromotive Force.**—Having now taken a given resistance as a standard of resistance, and the silver ampere as the standard current—the E.M.F. or voltage of a cell may be regarded as that which drives the current through a wire and against its resistance. From this point of view the E.M.F. of a cell is analogous to the difference of pressure which may be said to drive the water through the pipe. Practical engineers call this E.M.F. pressure, and this name has been preserved by the Board of Trade. Again, if we consider a part of a circuit when a current is passing through it, we may say that the current is due to something which is called "pressure." This point of view will be more fully considered under electrostatics. It appears to give a tangible idea of the phenomena, and is found satisfactory as a working hypothesis. When the poles of a Daniell cell are joined a current passes; if the poles are joined by a resistance of 1 ohm, and a current of 1 ampere passes—then the voltage is called 1 volt. If it is found that two amperes pass, then the voltage is 2 volts: and generally the E.M.F. is proportional to the current.

The Board of Trade definitions are—

(1) That the standard of electrical pressure (E.M.F.)

should be denominated the volt, being the pressure which, if steadily applied to a conductor whose resistance is 1 ohm, will produce a current of 1 ampere.

(2) That the electrical pressure (at a temperature of 62° F.) between the poles of the voltaic cell known as Clark's cell, prepared in accordance with the specification attached, may be taken as not differing from a pressure of 1.434 volts by more than one part in a thousand.

2. **The Simple Circuit: E.M.F., and Resistance of a Cell.**—If E is the E.M.F. of a cell, and i the current obtained when passing through a given conducting wire, then $E = i\bar{R}$ where $\bar{R}$ is the resistance of the whole circuit. This resistance is made up of the resistance of the external part of the circuit, and the resistance of the cell. That the cell exerts a resistance is proved from the fact that it becomes hot; and Joule has proved experimentally that heat is created in a cell according to the same law as in a metallic conductor. If the resistance of the external part of the circuit—the external resistance—is R, and the resistance of the cell—the internal resistance—is r, then the E.M.F. of the internal part of the circuit is ir, and of the external iR. Hence the E.M.F. of the whole circuit is $iR + ir$; i.e. $i(R + r)$, and this in the case of the constant cells is equal to the E.M.F. of the cell. Hence $E = i(R + r)$.

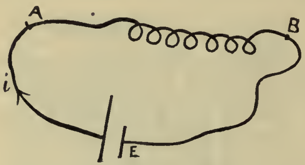


Fig. 52.

The student should carefully perform the following experiments. All that is required for the experiments is a piece of wire, the resistance of which is about $\frac{1}{3}$ of the ohm.

Expt. 1.—*To find the length of copper wire of a given gauge, the resistance of which will be equal to that of a Daniell cell, and to gain an idea of the comparative resistances of two different cells, and of two wires of different material.*

Measure the current from a Daniell cell.

Now insert such a length of the wire into the circuit until the current is reduced to half. To reduce a current to half, the tangent of the angle must be reduced to half. Since the current is halved, the resistance of the circuit must be doubled. Hence the resistance of the wire must be equal to that of the rest of the circuit: *i.e.* that of the cell, the galvanometer, and the connecting wires. To show that these last offer very small resistance, try the effect of putting a piece of wire of the same gauge as that of the galvanometer, and of length equal to that of the galvanometer and connecting wires together.

In a similar manner find the resistance of a Bunsen cell, in terms of the resistance of a length of wire of the same gauge as used for the Daniell. Hence compare the resistances of the two cells.

In a similar manner find what length of German-silver wire of same gauge as the copper wire is required to reduce the current of a Daniell cell to half. Hence compare the resistance of the German-silver wire and the copper wire. It will be found that the copper wire will be ten or eleven times the length of the German-silver wire of the same gauge.

The student may verify the fact that the temperature of the cell increases whilst the current is passing by placing a thermometer in the liquid.

EXPT. 2.—*To find the resistance of a Daniell cell in terms of the standard $\frac{1}{3}$ ohm; and then the E.M.F. in volts.*

(i) Connect up the cell through a tangent galvanometer and find the deflection d°. Then

$$E = i(B+G) = k \tan d \times (B+G). \qquad (1)$$

where

E = E.M.F. of cell.
B = resistance of cell.
G = resistance of galvanometer.

(ii) Now put the standard resistance R in the circuit, and measure the deflection d'. Then

$$E = i'(B+G+R) = k \tan d' \times (B+G+R) \qquad (2)$$

from (1) and (2)

$$k \tan d \times (B+G) = k \tan d' \times (B+G+R)$$
$$\therefore \quad (B+G)\{\tan d - \tan d'\} = R \tan d'$$
$$\therefore \quad B+G = \frac{R \, . \tan d'}{\tan d - \tan d'}.$$

If therefore G is very small, this is practically the resistance of the cell.

Find now the reduction factor of the galvanometer in amperes, and measure the current i in amperes, and since $(B+G)$ has been found in ohms

$$E = i(B+G), \text{ gives the E.M.F. in volts.}$$

Ex.—The deflection when a cell is connected directly through a galvanometer is 35°, when a resistance of 2 ohms is introduced the deflection is 8°, the reduction factor being 3 amperes.

Here

$$\begin{aligned}
&\text{(i)} \quad E = k \tan 35^\circ \times B \qquad \text{neglecting } G\\
&\text{(ii)} \quad E = k \tan 8^\circ \times (B+2)\\
\therefore\ & B \times \tan 35^\circ = (B+2)\tan 8^\circ\\
& B \times .574 = (B+2) \times .139\\
& \therefore\ B = .64\\
& \text{from (i) } E = 3 \times .574 \times .64\\
& \qquad = 1.1 \text{ volts.}
\end{aligned}$$

EXPT. 3.—*Find the resistance of a wire in terms of the standard.*

(i) Place the wire in the circuit of a cell and a galvanometer. The galvanometer should be chosen so that the deflection is neither too small nor too large. Read the deflection d.

Then

$$E = k \tan d(B+G+X) \qquad (1).$$

(ii) Replace the wire by a wire of known resistance R and measure the deflection d'

$$E = k \tan d'(B+G+R) \qquad (2).$$

(iii) Now replace the wire by a second wire of known resistance R′ and measure the deflection d''

$$E = k \tan d''(B+G+R') \qquad (3).$$

From (2) and (3) $(B+G)$ can be found,
and from (1) and (3) X can be found.

Note.—The resistances R and R′ should be such that the deflection d is between the deflections d' and a''.

Ex.—If in the example of the last experiment, an unknown resistance be placed in the circuit and the deflection found to be 25°, find the unknown resistance.

As in the previous example from the two equations find B or B + G. In this case $B = .64$.

Now the third equation is

$$E = k \tan 25^\circ (B + G + X) \qquad \text{or since}$$

in this case G can be neglected

$$\begin{aligned} E &= k \tan 25^\circ (B + X) \\ \therefore \quad &= k \times .423 (B + X) \\ \therefore \quad B \times .574 &= (B + X) \times .423 \\ X &= \frac{.151}{.423} \times B = \frac{151 \times .64}{423} = .23. \end{aligned}$$

Therefore the resistance $= .23$ ohms.

EXPT. 4.—*To compare the E.M.F.'s of a Daniell and a Bunsen cell by direct T.G. methods.*

Find the resistance B + G in each case. Then from the equations

$$\begin{aligned} E &= k \tan d \times (B + G) \\ E' &= k \tan d' \times (B' + G), \end{aligned}$$

we get

$$\frac{E}{E'} = \frac{\tan d \ (B + G)}{\tan d' \ (B' + G)}.$$

EXERCISES

I

1. The E.M.F. of a cell is 2 volts, and the resistance $\frac{2}{3}$ ohm, find the current when the external resistance is (i) $\frac{1}{3}$ ohm; (ii) 3 ohms; (iii) 100 ohms.

2. The E.M.F. of a cell is 2 volts, and internal resistance $\frac{1}{3}$ ohm, find the difference between the currents when the external resistance is 10 ohms (i) neglecting internal resistance; (ii) taking the internal resistance into account. What is the difference if the external resistance is 100 ohms?

3. A cell will give a deflection of 75° in a T.G. when the external resistance is negligible, and when a resistance of 2 ohms is put in the deflection is 22°. Find the resistance of the cell.

4. A Bunsen cell will give a deflection of 15°, and a Daniell cell a deflection of 5° in a T.G. Neglecting the resistances of the cells, compare the E.M.F.'s of the cells.

5. Find the resistance and E.M.F. of a cell from the following data:—

   When the cell is connected direct to the T.G. there is a deflection of 70°.

   When a resistance of 1 ohm is introduced the deflection is found to be 38°.45′.

   Radius of coils 15 cm., number of turns 5. H = .18.

6. Find the resistance of a coil from the following data :—

When the coil is in circuit, deflection is 40°.
When 2 ohms is in circuit, deflection is 30°.
When 1 ohm is in circuit, deflection is 45°.

7. What will be the deflection in Question 6 when 1.5 ohm is in circuit; and what resistance must be put in to give a deflection of 10°?

8. Show how to compare the E.M.F.'s of two cells.

9. The resistances of two cells are respectively .3 ohm and .9 ohm, and the resistance of the galvanometer is .1 ohm. The deflections are respectively 60° and 30°. Compare the E.M.F.'s of the cells.

II

1. What is the practical standard of resistance, and how is it related to the absolute ohm? What is the B.A. ohm? Express Siemens's unit in terms of the "standard ohm."

2. What is the standard current, and how is it related to the absolute ampere?

A current is measured by means of a tangent galvanometer, how would you determine its value in terms of the standard ampere?

3. Define the "standard volt," and describe the standard cell.

SECTION VIII

On Distribution of Current in Arc and in Series

1. **On the Law of Division of Current amongst the Branches of a Multiple Arc.**—If two or more conductors are arranged end to end so that the same current passes through each of them, they are said to be arranged in **series**. If, on the other hand, two or more conductors have one set of ends joined together, and the other set of ends also joined together, so as to form one conductor, the current dividing itself between the different wires or branches, they are said to be arranged in **multiple arc**, or **in parallel**. We have seen that if a wire connect the two poles of a cell, the current which will pass will depend upon the resistance of the wire, the greater the resistance the less the current. Now we may look at this from another point of view, and say that one wire or conductor **conducts the current** better than another, and we may speak of the **conductance** or conductivity of the wire—and it follows that the conductivity is inversely proportional to the resistance of the wire. The unit of conductivity is the conductivity of a conductor the resistance of

which is the ohm. This unit is called the **Mho**, and from the definition it follows that a wire the resistance of which is R ohms, has a conductance of $\frac{1}{R}$ mhos.

We would naturally expect that the resistance of two or more conductors arranged in series would be the sum of their resistances, and this will be proved later in this section.

We would also naturally expect that the conductance of two or more wires arranged in arc would be the sum of their conductances. This fact is suggested by the analogy of the flow of water in parallel channels, but it requires proof, and this proof will be given in Prop. 2 following. We would also expect by the same analogy that the current would divide itself between conductors in arc, in the ratio of their conductances, and this will also be proved correct.

It will be convenient to state these facts and apply them in simple cases before proceeding to the proofs and to more complex examples.

(1) The resistance of a number of conductors arranged in series is the sum of their separate resistances.

If R_1, R_2, R_3, ohms are the resistances of three conductors, and R ohms their combined resistance when arranged in series, then

$$R = R_1 + R_2 + R_3.$$

(2) The conductance of a number of conductors arranged in multiple arc, is the sum of their separate conductances.

If the resistances of three conductors arranged in arc are respectively R_1, R_2, R_3 ohms, then their conductances are $\frac{1}{R_1}$ $\frac{1}{R_2}$ $\frac{1}{R_3}$ mhos, and if R ohms is the equivalent resistance of the multiple arc, $\frac{1}{R}$ mhos is the conductance of the arc

$$\therefore \quad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$

(3) If a current passes through a multiple arc it will divide itself amongst the branches in the ratio of their conductances.

If x, y, z amperes are the currents in the branches R_1, R_2, R_3, then the currents will be given by

$$\frac{x}{\frac{1}{R_1}}=\frac{y}{\frac{1}{R_2}}=\frac{z}{\frac{1}{R_3}}.$$

If i is the total current then $i=x+y+z$,

$$\therefore \quad \frac{x}{\frac{1}{R_1}}=\frac{y}{\frac{1}{R_2}}=\frac{z}{\frac{1}{R_3}}=\frac{x+y+z}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}=\frac{i}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}.$$

It follows also that

$$xR_1=yR_2=zR_3=e \quad \text{say}$$

$$\therefore \quad x=\frac{e}{R_1} \quad y=\frac{e}{R_2} \quad z=\frac{e}{R_3},$$

$$\text{and} \quad i=\frac{e}{R_1}+\frac{e}{R_2}+\frac{e}{R_3}.$$

The above laws can be verified by direct experiment.

EXPT. 1.—*Find by the T.G. method of Section VII. the resistance of, say three wires, then find their resistance when in series and in arc, and verify the above laws.*

EXPT. 2.—*Take two T.G.'s of practically negligible resistance, and arrange them respectively in circuit with* 2 *ohms and* 3 *ohms, so as to form a multiple arc of* 2 *ohms and* 3 *ohms, with a T.G. in each branch. Now send a current through the arc by means of a Bunsen cell and measure the current, and show that these currents are in the ratio of the conductances* $\frac{1}{2}$, $\frac{1}{3}$ *mhos.*

EX. 1.—Three conductors of resistances 1, 2, 3 ohms are arranged in parallel, find their combined resistance.

The conductances are respectively $\frac{1}{1}$, $\frac{1}{2}$, $\frac{1}{3}$ mhos.

$\therefore$ total conductance $=1+\frac{1}{2}+\frac{1}{3}=\frac{11}{6}$ mhos.

$\therefore$ total resistance $=\frac{6}{11}$ ohm.

EX. 2.—If a current of 11 amperes flows into the multiple arc of Ex. 1, find the current in each branch.

Let x, y, z amperes be the currents then

$$\frac{x}{1}=\frac{y}{\frac{1}{2}}=\frac{z}{\frac{1}{3}}=\frac{x+y+z}{1+\frac{1}{2}+\frac{1}{3}}=\frac{11}{\frac{11}{6}}=6,$$

$$\therefore \quad x=6,\ y=3,\ z=2.$$

Ex. 3.—A cell of 2 volts E.M.F. and $\frac{1}{3}$ ohm resistance is used to drive a current through the multiple arc of Ex. 1, find the current through the cell, and through each branch.

1st. Find resistance of the arc.

By Ex. 1 this is $\frac{6}{11}$ ohms.

2nd. Find total resistance of the circuit.

This is $\frac{1}{3}+\frac{6}{11}=\frac{29}{33}$ ohm.

Hence the main current i amperes is given by

$$i=\frac{2}{\frac{29}{33}}=\frac{66}{29}$$

and

$$\frac{x}{1}=\frac{y}{\frac{1}{2}}=\frac{2}{\frac{1}{3}}=\frac{\frac{66}{29}}{\frac{11}{6}}=\frac{396}{319},$$

$$\therefore \quad x=\frac{396}{319}; \quad y=\frac{198}{319}; \quad z=\frac{132}{319}.$$

Ex. 4.—A cell of 2 volts E.M.F. is used to drive a current through a galvanometer of 40 ohms resistance, the terminals of the galvanometer being connected by a wire of 5 ohms resistance. If the resistance of the cell and connecting wires is 4 ohms, what is the current passing through the galvanometer?

(i) Find the current through the cell.

The galvanometer and the 5 ohms resistance form a multiple arc, the conductance of which is $\frac{1}{40}+\frac{1}{5}$, *i.e.* $\frac{9}{40}$ mhos.

Hence the resistance of the multiple arc is $\frac{40}{9}$ ohms.

$\therefore$ total resistance of the circuit is $4+\frac{40}{9}$, *i.e.* $\frac{76}{9}$ ohms.

$$\therefore \quad i=\frac{\mathrm{E}}{\mathrm{R}}=\frac{2}{\frac{76}{9}}=\frac{18}{76}=\frac{9}{38} \text{ amperes.}$$

(ii) And if x, y amperes are the currents through the galvanometer and the wire respectively

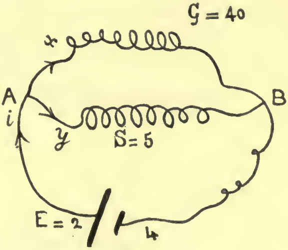


Fig. 53.

$$\frac{x}{\frac{1}{40}}=\frac{y}{\frac{1}{5}}=\frac{x+y}{\frac{1}{40}+\frac{1}{5}}=\frac{\frac{9}{38}}{\frac{9}{40}}=\frac{20}{19}.$$

$$\therefore \quad x=\frac{1}{40}\times\frac{20}{19}=\frac{1}{38}.$$

$\therefore$ current through the galvanometer $=\frac{1}{38}$ ampere.

Note.—If the current to be measured produces too great a deflection of the galvanometer needle, a part only of the current may be allowed to pass through the galvanometer by joining the poles of the galvanometer through a resistance. The current then, as in the above example, divides between this resistance and the galvanometer. Such a resistance, which is used to shunt part of the current through itself, is called a **Shunt.** The resistance of a shunt will depend upon the resistance of the galvanometer. The greater current required to pass through the galvanometer the greater must the resistance of the shunt be.

Ex.—A galvanometer has a resistance of 200 ohms, what must be the resistance of the shunt so that (1) $\frac{1}{10}$, (2) $\frac{1}{100}$ of the main current shall pass through the galvanometer?

(1) Here $$x=\frac{i}{10},$$

$$y=\frac{9i}{10}$$

and $$\frac{x}{\frac{1}{200}}=\frac{y}{\frac{1}{S}}$$

if S ohms is the resistance of the shunt.

$$\therefore \quad 200x=Sy,$$

$$\text{or} \quad 200\times\frac{i}{10}=S\times\frac{9i}{10}.$$

$$\therefore \quad S=\frac{200}{9}, \quad \textit{i.e.}\ \frac{1}{9}\ \text{of the galvanometer resistance.}$$

(2) In the second case—

$$Sy=200x$$

$$x=\frac{i}{100}$$

$$y=\frac{99i}{100}.$$

$\therefore$ current must divide in ratio of 99 to 1.

$\therefore$ resistance must be in ratio of 1 to 99.

$\therefore$ resistance of shunt $=\frac{1}{99}$ resistance of the galvanometer.

2. Theoretical Proof of the Laws of Divided Circuits.

PROP. 1.—*To find the resistance of two or more conductors arranged in series*, i.e. *end to end.*

Let AB, BC, CD be three conductors arranged in series, so that the same current passes through each of them.

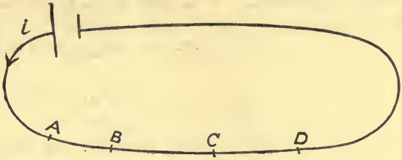


Fig. 54.

Let e_1, e_2, e_3 be the E.M.F. of the conductors, and R_1, R_2, R_3 their resistances; and let i be the current passing through them.

Then

$$e_1 = iR_1 ; \quad e_2 = iR_2 ; \quad e_3 = iR_3 .$$

∴ The E.M.F of the conductor AD

$$= e_1 + e_2 + e_3 = iR_1 + iR_2 + iR_3$$
$$= i(R_1 + R_2 + R_3).$$

If R is the equivalent resistance then the E.M.F. $= iR$.

$$\therefore \quad R = R_1 + R_2 + R_3 .$$

i.e. **The resistance of a number of conductors in series is the sum of their separate resistances.**

PROP. 2.—*To find the resistance of two or more conductors arranged in multiple arc (or in parallel*), i.e. *with one set of ends joined together to form one terminal and the other set of ends joined to form the other terminal.*

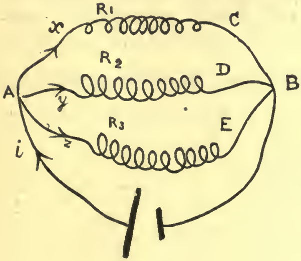


Fig. 55.

Let ACB, ADB, AEB be three conductors arranged in arc; and suppose their resistances are respectively R_1, R_2, R_3 and let i be the current entering at A.

Suppose the current divides, so that the current in ACB is x; in ADB is y; and in AEB is z.

Then

$$i = x + y + z.$$

Also the E.M.F. between A and B is the same whatever branch is taken. Let this E.M.F. be e.

$$\therefore \quad e = x\mathrm{R}_1, \quad \text{or } x = \frac{e}{\mathrm{R}_1},$$

$$e = y\mathrm{R}_2, \quad \text{or } y = \frac{e}{\mathrm{R}_2},$$

$$e = z\mathrm{R}_3, \quad \text{or } z = \frac{e}{\mathrm{R}_3}.$$

$$\therefore \quad i = x + y + z = \frac{e}{\mathrm{R}_1} + \frac{e}{\mathrm{R}_2} + \frac{e}{\mathrm{R}_3},$$

If R is the resistance of a conductor equivalent to the multiple arc between A and B,
then

$$e = i\mathrm{R}, \quad \text{or } i = \frac{e}{\mathrm{R}}.$$

$$\therefore \quad \frac{e}{\mathrm{R}} = \frac{e}{\mathrm{R}_1} + \frac{e}{\mathrm{R}_2} + \frac{e}{\mathrm{R}_3},$$

or

$$\frac{1}{\mathrm{R}} = \frac{1}{\mathrm{R}_1} + \frac{1}{\mathrm{R}_2} + \frac{1}{\mathrm{R}_3}.$$

But the reciprocal of the resistance of a conductor is the measure of the conductance of the conductor.

Hence the above result may be stated as follows :—

The conductance of a series of conductors in multiple arc is the sum of their separate conductances.

Note.—If R is found as above, then the currents in the separate branches are given by

$$x = \frac{e}{\mathrm{R}_1} = \frac{i\mathrm{R}}{\mathrm{R}_1},$$

$$y = \frac{e}{\mathrm{R}_2} = \frac{i\mathrm{R}}{\mathrm{R}_2},$$

$$z = \frac{e}{\mathrm{R}_3} = \frac{i\mathrm{R}}{\mathrm{R}_3}.$$

That the E.M.F. between A and B is the same for each branch is evident from the fact that the current will adjust itself in order that the work done in each branch per coulomb of electricity may be the same, otherwise more work would be done in one circuit than in the other when unit quantity of electricity passes.

The equation $i=x+y+z$ follows from the ordinary law of continuity, and definition of quantity of electricity.

Both these facts—

(i) The E.M.F. is the same for each branch,

(ii) The current in the main wire is equal to the sum of the currents in the multiple arc,

may be verified by placing copper voltameters in the main circuit, and in each of the branches.

Ex. 1.—A current from a cell of E.M.F. = E is sent through a galvanometer of resistance R ohms, the terminals of which are joined by a wire of S ohms: find the current through the galvanometer, the resistance of cell and connecting wire being R′ (see Fig. 53).

Let i be the current in the cell.

Resistance of the multiple arc $=\frac{\mathrm{RS}}{\mathrm{R+S}}$.

If x is the current in the galvanometer, then

$$x\times \mathrm{R}=i\times\frac{\mathrm{RS}}{\mathrm{R+S}}.$$

$$\therefore\quad x=\frac{\mathrm{S}}{\mathrm{R+S}}\cdot i,$$

i.e. of the total current i, only the fraction $\frac{\mathrm{S}}{\mathrm{R+S}}i$ passes through the galvanometer.

The total resistance of the circuit $=\mathrm{R}'+\frac{\mathrm{R\,.\,S}}{\mathrm{R+S}}$.

$$\therefore\quad i=\frac{\mathrm{E}}{\mathrm{R}'+\frac{\mathrm{R\,.\,S}}{\mathrm{R+S}}}.$$

$$\therefore\quad x=\frac{\mathrm{S}}{\mathrm{R+S}}\times\frac{\mathrm{E}}{\mathrm{R}'+\frac{\mathrm{R\,.\,S}}{\mathrm{R+S}}}=\frac{\mathrm{S\,.\,E}}{\mathrm{R}'(\mathrm{R+S})+\mathrm{R\,.\,S}}.$$

Note.—If R′ is very great compared with either R or S, then $x=\frac{\mathrm{SE}}{\mathrm{R}'(\mathrm{R+S})}$.

EXERCISES

1. Find the resistance of—

   (*a*) 2, 3, 5 ohms in arc;
   (*b*) .2, .3, .5 ohms in arc;
   (*c*) 2 and 3 ohms arranged in series with 5 and 6 in arc.

2. Find what resistances respectively must be joined in arc with 1.1, 1.01, 1.02 ohms to give a resistance of 1 ohm.

3. Given 1, 2, 3, ohms find the resistances which may be obtained by joining one or more in series or arc, or a combination of series and arc.

4. A conductor is formed of 2 ohms arranged in series with 3 and 5 ohms in multiple arc; another is formed by joining the multiple arc of 1 and 2 ohms in series with 3 ohms. Find the resistance of these two conductors arranged in arc.

5. A current of 5 amperes is sent through a resistance consisting of 2, 3, 5 ohms arranged in arc: find the current in each conductor.

6. A current of .3 ampere is sent through a multiple arc consisting of 30 ohms and .5 ohm: find the current in each branch.

7. Find the current obtained from a cell of 2 volts, and internal resistance 3 ohms, external resistance an arc of 6 and 4 ohms: and find the current in each branch.

8. A cell sends a current of .1 ampere through a galvanometer of 30 ohms resistance shunted with a 2 ohm shunt: find the current through the cell.

9. The current through a galvanometer alone gives a deflection of 60°, but shunted with 2 ohms it gives a deflection of 10°. Find the resistance of the galvanometer.

10. A battery of 5 ohms resistance sends a current (1) through a galvanometer of 10 ohms resistance direct (2) through the galvanometer shunted with 4 ohms. Compare the main currents, and the currents in galvanometer in each case.

SECTION IX

Specific Resistance

EXPT. 1, § 7 shows that the resistance of German-silver wire is about ten times greater than that of a piece of copper wire of same length and diameter; and the experiments performed will have shown that the resistance of a given length and diameter of a substance depends upon the nature of the substance. This property of bodies is called specific resistance.

The specific resistance of a substance is measured by the resistance of a cube of it, the edge of which is 1 cm., the current passes from one face to the opposite face.

The specific resistance is measured in absolute measure by the resistance of a wire 1 cm. long, 1 sq. cm. sectional area, *i.e.* in C.G.S. units per cm. sq. cm.

Since a cm. length is very short, and a square cm. sectional area very large, the specific resistance of ordinary metallic conductors will be very small. Now the absolute unit of resistance is one thousand millionth of an ohm, and hence specific resistance of conductors will be conveniently expressed in absolute units per cm. sq. cm. The specific resistance of German-silver wire is about 21176 absolute units, *i.e.* 21176×10^{-9} ohms per cm. sq. cm. The specific resistance of glass when cold is very great, about 2240×10^{6} ohms per cm. sq. cm.

The meg.-ohm = 1 million ohms
The micr.-ohm = 1 millioneth of an ohm.

Hence the specific resistance of the glass = 2240 meg.-ohms per cm. sq. cm., and the specific resistance of G.S. = 21.176 micr.-ohms per cm. sq. cm.

PROP.—*To prove that the resistance of a wire of uniform sectional area varies directly as its length and inversely as its area.*

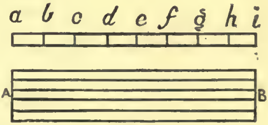


Fig. 56.

Let σ be the specific resistance of the material.

Suppose the length of the wire is l cm.

Arrange "l" unit cubes of the material in a line end on, as in figure. Then by Prop. 1 (§ 8) the resistance of the cylinder thus formed is the sum of the separate resistances, *i.e.* since the resistance of each cube is σ, resistance of a wire 1 sq. cm. section, and l cm. long is $l\sigma$.

Suppose the sectional area is a sq. cm.

Arrange a cylinders side by side, so as to form one wire l cm. long, and a sq. cm. sectional area. The resistance of one of these is $l\sigma$, and $\therefore$ by (Prop. 2, § 8) resistance of a of them arranged in parallel is $\frac{l\sigma}{a}$.

$$\therefore \quad \text{The resistance of the wire is } \frac{\sigma l}{a}.$$

EXPT. 1.—*To find approximately the specific resistance of G.S.*

Take about six metres of the wire, and carefully measure its length and its diameter (the diameter should be measured in six different parts of the wire and the mean taken).

Then measure the resistance of the wire in ohms.

Let R ohms be the resistance
l cm. the length
d cm. the thickness

$$\text{Then the sectional area } = \frac{\pi d^2}{4}$$

and

$$R = \frac{l\sigma}{a} = \frac{4l\sigma}{\pi d^2} \quad \therefore \sigma = \frac{\pi d^2 R}{4l}.$$

This gives the specific resistance in ohms per cm. sq. cm.

Ex. 1.—Find the resistance of a wire 100 cm. long; and .1 mm. diameter; specific resistance 21176 C.G.S.

Hence radius of cross section = .005 cm.

$$\therefore \quad a = 3.1416 \times (.005)^2$$

$$\therefore \quad \text{Resistance} = \frac{\sigma l}{a} = \frac{21176 \times 100}{3.1416 \times (.005)^2}$$

$$= \frac{21176}{3.1416 \times 25} \times \frac{100}{.000001}$$

$$= 2.7 \times \frac{10^2}{10^{-7}} = 2.7 \times 10^9$$

$$\therefore \quad \text{Resistance} = 2.7 \text{ ohms}$$

Ex. 2.—Find the diameter of copper wire ($\sigma = 1642$ C.G.S.) so that 3 metres may have a resistance of 2 ohms.

$$\text{Resistance} = \frac{\sigma l}{a} = 2 \times 10^9$$

and if

$$r = \text{radius in cm.}$$

$$a = \pi r^2 \quad l = 300$$

$$\therefore \frac{1642 \times 300}{\pi r^2} = 2 \times 10^9$$

$$r^2 = \frac{1642 \times 300}{3.1416 \times 2 \times 10^9}$$

$$r = \sqrt{\frac{1642 \times 300}{3.1416 \times 2 \times 10^9}}$$

$$= .008854 \text{ cm.}$$

$$\therefore \quad \text{diameter} = .177 \text{ mm.}$$

Ex. 3.—A copper wire occupies a volume of v c.c., and has a resistance of R C.G.S. units, what must its length and sectional area be?

Here

$$R=\frac{\sigma l}{a}$$

and

$$v=la$$

$$\therefore\ l^2=\frac{Rv}{\sigma}\quad l=\sqrt{\frac{Rv}{\sigma}}$$

and

$$a^2=\frac{v\sigma}{R}\quad a=\sqrt{\frac{v\sigma}{R}}$$

If D is the density then

$$\text{Mass}=v\text{D gm.}$$

EXERCISES

1. Find resistance of (*a*) 1 metre of G.S. wire 1 mm. diameter.
(*b*) 1 metre of G.S. wire .1 mm. diameter.
(*c*) 1 metre of copper wire .16 mm. diameter.

2. What length of copper wire (16 B.W.G.), *i.e.* 1.651 mm. diameter, must be taken to give a resistance of .3 ohm? also find what length of same copper wire (22 B.W.G.) .7112 mm. diameter must be taken to have the same resistance.

3. A mil is one-thousandth of an inch. If the resistance of one foot of copper wire, 1 mil-diameter, is 10.233 ohms, find in C.G.S. units the specific resistance of the copper.

4. The density of copper is 555 lbs. per cubic foot, find the resistance of 2 lbs. of copper wire of 22 B.W.G. (.7112 mm. diameter).

5. Express the specific resistance of copper in ohms per square mil-mile.

6. A galvanometer has 2000 turns of copper wire of mean radius 3 cm. and .305 mm. diameter, find the resistance of the galvanometer.

7. Find the length of G.S. wire of the following gauges, which must be taken to form a resistance of 1 ohm:—

7.62 mm.; 4.2 mm.; 1.473 mm.; .635 mm. diameter respectively.

8. Find the resistance in ohms of 1 metre of G.S. wire of the gauges in Question 7.

SECTION X

On a Battery of Cells

1. On the E.M.F. and Resistance of a Cell.—The E.M.F. of a cell depends only upon the materials of which it is constructed **and not upon its size.** Whatever the size of a Daniell cell may be, whether a pint or quart or gallon, the same mass of zinc enters into action per coulomb (by Faraday's Laws of Electrolysis), and if the materials are the same, the same quantity of energy will be set free by the combustion of the zinc; and hence the E.M.F. will be the same. The size of the cell will affect the rate at which the zinc is consumed, *i.e.* affect the rate at which the energy is set free, and this it does in virtue of its resistance. Again, a large cell will give the same current for a longer time than a smaller cell, because it will probably contain more zinc to be consumed; it has a greater storage of energy, but since, in the *same kind* of cell the same amount of work is done per E.C.E. of zinc, *i.e.* per coulomb, the E.M.F. is the same for cells of all sizes.

If the external resistance is very small compared with the resistance of each cell, the large cell will give a greater current, because its resistance is less; and in this case the resistance of the cell is the main resistance. If, on the other hand, the external resistance is large compared with the resistance of the cell, then the current obtained would be the same, or nearly the same, whichever cell is used.

EXPT. 1.—*Compare the resistance of a Daniell cell when filled with its resistance when about half filled with the liquids.*

Set up a Daniell cell with the vessels one-half full of the liquids, and measure its resistance by Expt. 2, § 7.

Now fill up the vessels and again measure the resistance of the cell.

EXPT. 2.—*Measure the current from a Daniell cell through about* 100 *ohms resistance;* (i) *when cell is full;* (ii) *when cell is less than a quarter full.*

It will be found that the current is practically the same in each case.

2. **On the E.M.F. of Cells in Series or in Arc.**—If two cells of equal E.M.F. be joined together in multiple arc (*i.e.* the two zinc poles joined to form one pole, and the two copper poles joined to form the other pole of the battery), the E.M.F. of the combination will be the same as the E.M.F. of one of them. For when one unit of electricity passes through the external resistance, a part will pass through one cell, and

the remaining part through the other, so that in the two cells together the E.C.E. of zinc will enter into action, and therefore E ergs. of work will be done, E being the E.M.F. of each cell.

In the same way, if any number of cells of equal E.M.F. be arranged in arc, the E.M.F. of the battery thus formed will be the same as the E.M.F. of one of them.

The resistance of a battery of cells arranged in arc can be found in the usual way. If there are n cells arranged in arc, the resistance of each being r ohms, then the resistance of the battery is $\frac{r}{n}$ ohms. If their resistances are different the resistance of the battery must be found by means of Prop. 2, § 8.

If two cells are arranged in series, so that the same current passes through each of them, the E.M.F. of the battery formed will be the sum of the separate E.M.F.'s of the cells. For when one unit of electricity passes through the external circuit, one unit will pass through each cell, and if it passes from zinc to copper in each case, then E.C.E. of zinc enters into chemical action in each cell. If E, E' are the E.M.F.'s respectively of the cells, then the total work done by the battery per unit quantity of electricity is $E + E'$. Hence the E.M.F. of the battery $= E + E'$.

Hence if any number of cells are arranged in series, the E.M.F. of the battery is the sum of the E.M.F.'s of the cells.

If two cells are arranged so that they oppose each other, then the current will be in the direction of the current due to the cell with the greater E.M.F. Let E' be the greater E.M.F., and E the smaller E.M.F. Then when unit quantity of electricity passes through the external circuit, E.C.E. of zinc will be consumed in the first cell; and the E.C.E. of copper will be consumed in the second case. The first cell will supply E' ergs. per coulomb; the second will absorb E ergs. per coulomb; so that $E' - E$ ergs. per coulomb is the energy set free for the circuit. Hence E.M.F. $= E' - E$.

3. On arranging a Battery of Cells.

PROP. 1.—*To find the current in the external circuit from a battery of similar cells, of equal E.M.F. (e) and resistance (r), arranged (s) in series and (n) in arc; the external resistance being R.*

The total E.M.F. $= s \,.\, e$.

The resistance of a row of cells in series is $s \,.\, r$,

and $\therefore$ the resistance of n such rows in arc $= \dfrac{s \,.\, r}{n}$.

Hence total resistance of circuit $= \mathrm{R} + \dfrac{s \,.\, r}{n}$.

$$\therefore \quad i = \frac{s \,.\, e}{\mathrm{R} + \dfrac{s \,.\, r}{n}}.$$

PROP. 2.—*To find the best way of arranging N cells, so as to obtain as large a current as possible.*

Suppose the battery is arranged s in series and n in arc, where $\mathrm{N} = s \,.\, n$,
then

$$i = \frac{s \,.\, e}{\mathrm{R} + \dfrac{s \,.\, r}{n}},$$

or dividing numerator and denominator by s

$$i = \frac{e}{\dfrac{\mathrm{R}}{s} + \dfrac{r}{n}}.$$

In order that this should be as great as possible $\dfrac{\mathrm{R}}{s} + \dfrac{r}{n}$ must be as small as possible.

Now

$$\left(\frac{\mathrm{R}}{s} + \frac{r}{n}\right)^2 = \left(\frac{\mathrm{R}}{s} - \frac{r}{n}\right)^2 + \frac{4 \,.\, \mathrm{R}}{s} \,.\, \frac{r}{n}$$
$$= \left(\frac{\mathrm{R}}{s} - \frac{r}{n}\right)^2 + \frac{4 \,.\, \mathrm{R} \,.\, r}{\mathrm{N}}.$$

Since the term $\dfrac{4 \,.\, \mathrm{R} \,.\, r}{\mathrm{N}}$ is always the same whatever s and n may be $\left(\dfrac{\mathrm{R}}{s} + \dfrac{r}{n}\right)^2$ is least when $\dfrac{\mathrm{R}}{s} - \dfrac{r}{n} = 0 \quad \therefore \dfrac{\mathrm{R}}{s} + \dfrac{r}{n}$ is least and $\therefore i$ is greatest when $\dfrac{\mathrm{R}}{s} - \dfrac{r}{n}$ is made as small as possible, *i.e.*

when $\frac{R}{s}$ is made as nearly equal to $\frac{r}{n}$ as possible, *i.e.* when R is made as nearly equal to $\frac{s \cdot r}{n}$ as possible.

Hence to obtain the largest current make the internal resistance $\frac{r \cdot s}{n}$ as nearly equal to the external (R) as possible.

Ex. 1.—20 cells. $r=2$, $R=20$.

Internal resistance, $\frac{2 \cdot s}{n}$.

$\therefore \frac{2 \cdot s}{n}$ should be as nearly equal to 20 as possible. Given that

$$s \cdot n = 20$$

$$\text{or } n = \frac{20}{s}$$

$$\therefore \frac{2s}{\frac{20}{s}} = 20$$

$$\therefore s^2 = 200$$

$$s = 14.3.$$

The two possible values of s are 20 and 10, and the corresponding values of n are 1 and 2.

In 1st case

$$\frac{R}{s} = \frac{20}{20} = 1$$

$$\frac{r}{n} = \frac{2}{1} = \frac{1}{2}$$

2nd case

$$\frac{R}{s} = \frac{10}{20} = \frac{1}{2}$$

$$\frac{r}{n} = \frac{2}{2} = 1.$$

Hence the difference $\frac{R}{s} - \frac{r}{n}$ is the same in each case.

Hence the greatest current will be obtained either by 20 cells in series or 10 in series and 2 in arc.

Exercises

1. A Daniell cell gives a deflection of 20°, part of the liquid is then poured out, and the deflection is then 10°. Compare the resistance of the cell in each case neglecting external resistance.

2. Two cells in series give a deflection of 60°, but opposed to each other only a deflection of 10°, compare their E.M.F.'s.

3. Two cells in series give a deflection of 60°, but opposed to each other the deflection is 30°. Compare their E.M.F.'s.

4. A Daniell cell with no external resistance gives a deflection of 15°, its resistance being .6 ohm. A Bunsen cell gives a deflection of 47°, its resistance being .3 ohm. Compare their E.M.F.'s.

5. The resistance of a Daniell cell is 1 ohm and external resistance 1 ohm, and the deflection is 10°. The resistance of a Bunsen cell is .5 ohm, and external resistance negligible, and deflection 60°.45′. Compare their E.M.F.'s.

6. Show how to arrange 100 cells of E.M.F. 2 volts, and internal resistance of 1 ohm to give the greatest current through (1) 100 ohms, (2) 50 ohms, (3) 25 ohms, (4) 20 ohms, (5) 5 ohms., (6) 2 ohms, (7) .1 ohm, (8) .01 ohm, (9) .001 ohm, and find the current in each case.

7. What is the best arrangement, for greatest current, of 20 cells, each of 8 ohms resistance, through 4 ohms external resistance?

EXAMPLES ON CHAPTER II

1. A piece of copper and a piece of zinc are put side by side into a vessel of dilute sulphuric acid. What takes place *in the vessel* when the copper and zinc are joined by a metal wire?

2. Give a drawing of a galvanic cell of copper, zinc, and dilute sulphuric acid, showing in what direction the positive current passes through a wire connecting the two metals, and also through the dilute sulphuric acid.

3. A piece of copper and a piece of zinc are put side by side into a vessel of dilute sulphuric acid. When a wire is placed so as to connect the copper and zinc, what new properties does the wire gain different from what it had before?

4. What is meant by a two-fluid cell? What is its advantage over a single-fluid cell?

5. If you were asked for a report on the merits of a primary battery, consisting of two metals immersed in a liquid, what observations would you make, and how would you make them?

6. What is the exact construction of a Daniell cell? Why is it called "constant"?

7. What actions, chemical and electrical, are to be observed in a galvanic cell made of ordinary zinc and copper plates, dipped in acidulated water?

Why does such a cell fail?

What is the object of amalgamating the zinc plates in a cell?

8. Describe the construction and action of a Grove cell. What is meant by Polarisation? How is it avoided in a Grove cell?

9. Describe the condition of a Daniell cell before the terminals are joined up and the action which takes place after they are joined.

10. How is a tangent galvanometer made? Give a proof of the law of its deflection, and state for what class of measurements this instrument is adapted.

11. Show that the deflection of a tangent galvanometer is independent of the strength of the galvanometer magnet.

12. Having given the horizontal intensity of the earth's magnetic field, give a detailed account of the construction of a tangent galvanometer by which the intensity of electric currents might be determined in absolute measure.

13. Describe a tangent galvanometer, and show how you would estimate a current by means of it, the value of $H = .18$.

14. In what direction is a tangent galvanometer placed relative to the magnetic meridian? Show by a sketch in what direction the magnetic needle is deflected by a current in a given direction.

15. What are the laws of "electrolysis"? Show how a "voltameter" may be used to measure the mean strength of a current.

16. A vertical partition of porous earthenware is fitted into a tumbler, and dilute sulphuric acid is poured into each compartment. Rods of common zinc and copper are placed respectively in the two compartments and connected by a wire. State what will be observed with regard to the evolution of gas, and how the observed phenomena will be modified when copper sulphate solution is poured into the compartment containing the copper rod.

17. You have access to the terminal wires made of copper of a hidden battery. Explain how you would tell which wire was connected with the zinc and which with the platinum pole of the battery by observing what happens when the ends of the wires are dipped at the same time into the same vessel containing a solution of sulphate of copper (cupric sulphate).

18. A piece of zinc and a piece of copper are each carefully weighed; they are then connected by a copper wire and dipped side by side into dilute sulphuric acid contained in an earthenware jar. After, say, half an hour, the pieces of zinc and copper are taken out of the acid, washed and dried, and weighed again. Would the weights be the same as at first? if not, how, and why, would they differ?

19. The electrochemical equivalent of zinc is ·000341. Find how many grammes of zinc would be burnt in 12 hours by 10 Bunsen cells joined up in series and working through an external resistance of 400 ohms, the electromotive force of a cell being 1.8 volts and its resistance ·36 of an ohm.

20. I wish to deposit 3 oz. of copper by electrolysis; and I have at my disposal three battery cells in which zinc is the positive plate. Find how much zinc is dissolved altogether, if I arrange the three cells in series. [$Cu = 63$: $Zn = 65$, both are dyads.]

21. Two copper wires, one connected with one terminal of a voltaic battery and the other connected with the other terminal, dip side by side, but without touching each other, into a solution of sulphate of copper. What happens to the immersed part of each wire?

22. What are the quantitative laws of electrolysis? A current of ·5 amperes was passed through an electrolytic cell containing a solution of a salt of silver; the electrochemical equivalent for silver is ·001134. How much silver was deposited in 10 min.?

23. A battery of 8 cells, connected in series, is used to decompose water in a voltameter; the chemical equivalent of zinc being 32·5 times

that of hydrogen, show how much zinc is consumed in the whole battery, while 1 grain of hydrogen is liberated in the voltameter.

24. What do you understand by the polarisation of the platinum plates employed as electrodes in a voltameter? How would you show experimentally in which direction the polarisation acts?

25. A single cell in which zinc is the positive plate, is used to effect the silver-plating of a spoon, by the usual process. How much zinc will have been dissolved in the cell when 3 gms. of silver have been deposited on the spoon?

[Given Zn = 65 divalent : Ag = 108 monovalent.]

26. What is meant by "electromotive force"? How could you tell whether the E.M.F. of one cell were greater or less than that of another?

27. On what does the quantity of heat generated in a conductor bearing a current depend? How would you support experimentally the accuracy of your statements?

28. Explain how the absolute electromotive force of a galvanic cell can be calculated from the heat of the chemical action that takes place in it when a current is passing.

29. How could you boil water by means of the current from a voltaic battery? Give a sketch to explain the arrangement of apparatus you would use.

30. What is meant by the electrical resistance of a conductor?

31. State and explain the meaning of Ohm's Law. There are in series four cells, each of two volts E.M.F., and $1\frac{1}{2}$ ohms resistance. The circuit is completed by a wire of 10 ohms resistance. Find the current that flows.

32. What is the unit used in practice of each of the three quantities occurring in the expression of Ohm's Law?

33. 40 cells of 1.8 volts E.M.F. each, have an internal resistance of three ohms each, the cells are joined up in series, the external conductor has a resistance of 60 ohms, what is the strength of the current in amperes?

34. A closed circuit contains a battery of one ohm resistance, a reflecting galvanometer of four ohms, and other conductors of two ohms resistance. The deflection of the galvanometer is 100 divisions of the scale. What will the deflection be (assuming it to be proportional to the strength of the current) when the terminals of the galvanometer are connected by a wire of four ohms resistance?

35. A current is reduced from 1.05 amperes to .77 by interpolating a resistance of 56 ohms: find the E.M.F.

36. How is it that if the poles of a battery are connected by a long thin wire, the battery does not get so hot as when a short thick wire is used?

37. Explain some method of determining the resistance of a wire.

38. A wire has a resistance of 15 ohms; it is then drawn until its diameter is halved; determine to what extent the resistance is altered by the alteration in shape.

39. The specific resistance of iron is 9827 and that of copper 1642 C.G.S. units; compare the thicknesses of iron and copper which have the same resistance per mile.

40. If the E.M.F. of a Grove cell is 1.7 volts, and its internal resistance

is .25 ohm, what current can it supply in an external circuit of resistance 3 ohms?

41. The terminals of a battery formed of seven Daniell cells in series are joined by a wire 35 feet long. One binding screw of a galvanometer is joined by a wire to the copper of the third cell (reckoned from the copper end). With what point of the 35 feet wire can the other screw of the galvanometer be connected so that the needle shall not be deflected?

42. What is the resistance of a wire in which a current of 3 amperes flows when an E.M.F. of 51 volts is maintained between its terminals?

43. The difference of potential between the poles of a battery of 10 ohms resistance is 3 volts before they are joined together. What is their difference of potential when joined together by a wire of 50 ohms resistance?

44. A Smee cell, when joined up with a tangent galvanometer whose resistance could be neglected, produced a deflection the value of whose tangent was 2.328. When resistances of .775 and 2.325 ohms respectively were successively introduced into the circuit the corresponding values were 0.966 and 0.445. Calculate from these data two values of the internal resistance of the cell to four places of decimals of an ohm.

45. Give the theory of Wheatstone's bridge, and state how you would proceed to determine the specific resistance of the substance of a wire.

46. How does the resistance of a conductor depend upon its dimensions?

47. Two copper wires, one of which is four metres long and weighs 7.5 grammes, and the other five metres long and weighs 4.2 grammes, are joined in "multiple arc" (that is, so that both wires connect the same two points) in a circuit in which a current of total strength 8.09 (amperes) is passing. The current will divide so that part passes through each wire; what will be the strength of the current conveyed by each?

48. The poles of a cell are connected by two wires, of 50 ohms and $2\frac{1}{2}$ ohms respectively, in multiple arc. Compare the currents in each wire and in the cell.

49. The resistances of two wires made of the same metal, *a* and *b*, are as 2 to 3. What are the relative amounts of heat developed in the wires —(1) when they are fastened end to end and the same current passes through them; and (2) when they are arranged in "multiple arc," that is, when each connects the ends of the same battery so that the battery current is divided between them?

50. Two cells, whose internal resistances are each 3 ohms, are connected in one case in series by a wire of 3 ohms resistance, and in the second are joined abreast and connected by the same wire. Compare the total heat produced in the two cases, and its distribution between battery and wire.

51. Ten voltaic cells, each of internal resistance 3 and electromotive force 4, are connected—

(*a*) in a single series;

(*b*) in two series of five each, the similar ends of each series being connected together,

If the terminals are in each case connected by a wire of resistance 20, show what is the strength of the current in each case.

52. Twenty-four cells, the E.M.F. of each of which is 2 volts, will drive a current of an ampere through an external resistance of 16 ohms

when arranged in series ; show that they can be otherwise arranged so as to drive the same current through the same resistance.

53. What is meant by connecting battery cells (1) in multiple arc, (2) in series?

54. An electric bell whose resistance is 25 ohms is to be worked by battery cells ; state, with reasons, what cells you would choose for the purpose and whether you would connect them in multiple arc or in series.

55. One hundred cells, each having an internal resistance of 4 ohms, are to be used with an external resistance of 25 ohms. How must the cells be connected up to give the strongest current, and what will be the current strength? The E.M.F. of each cell is 1.8 volt.

56. A current of 0.016 amperes is to be sent through a resistance of 360 ohms. What is the smallest number of Leclanche's cells, each with E.M.F. 1.4 and the resistance 15 ohms, by which this could be effected?

What would be the maximum strength of the current if double the number of such cells were used.

57. Four cells, the resistance of each of which is .8 ohm and E.M.F. 1.2 volt, are joined up (1) in series, (2) in multiple arc. Find the strength of the current in each case, the resistance of the interpolar wire being 4.8 ohms.

58. There are twelve cells, each of $1\frac{1}{2}$ volts E.M.F. and $\frac{1}{3}$ ohm resistance. They are arranged in pairs; in each pair the zincs are connected with the zincs and the coppers with the coppers. Then these six double cells are arranged in series. What current will be driven through an external resistance of 3 ohms?

If the external resistance had been 50 ohms, how would you have arranged the cells and why?

CHAPTER III

ELECTROSTATICS

SECTION I

EXPT. 1.—Take a glass rod and rub it with a silk pad, and then suspend it in a paper stirrup hung from a dry silk thread. Now rub another glass rod, and present it to one end of the suspended rod. It will be found to repel each end, whatever part of the rod is presented to it. Now rub a vulcanite rod with the rubber and present it to the suspended glass rod; it will be found to attract each end of the glass rod whatever part of it is presented to the rod.

Both the rods have, by the rubbing, been endowed with properties in virtue of which they attract light particles of matter. They are said to be endowed with electricity. Now the rubbed glass rods repel each other, but a rubbed vulcanite rod attracts a rubbed glass rod, hence they are endowed with different kinds of electricity.

LAW 1.—Bodies endowed with like electricity repel each other, bodies endowed with unlike electricity attract each other.

This property, which appears to be given to bodies by rubbing them, was early known to the ancients, and was first discovered to be possessed by amber (*ἤλεκρον*) when rubbed by woollen material. The property was called electric, and the body was supposed to be coated with a fluid, to which the name electrical fluid was given. There is now every reason to suppose that the cause of the attraction and repulsion rests in a change in the nature of the medium surrounding the bodies, and that this change in the nature of the medium is made manifest at the point where the medium abuts on to other bodies. At any rate there is a condition or state manifested on the surface of bodies, and the discussion of the laws which this state is found to obey forms the subject of electricity. In calling this manifestation electricity, we avoid committing ourselves to any statement as to its real nature. A body which has been endowed with this property —called electrical property—is said to be "charged with electricity." A spring gun may be said to be charged when the spring is compressed.

EXPT. 2.—(*a*) Take a brass rod and rub it with the rubber, and test it for electrical property. It will be found neither to attract light matter nor to attract or repel the suspended glass rod. The rubber, however, will be found to attract light matter, and also to act on the glass rod. Now push the brass rod into a vulcanite handle, and then rub the brass rod, taking care not to touch the brass with any part of the body. The brass rod will now be found to possess electrical properties, or in other words, to be endowed or charged with electricity. Now touch the brass rod, and its electrical property will disappear.

(*b*) Take a metal plate and place it on glass supports. Rub a sheet of vulcanite and place it on the plate, and take it away. It will be found that the metal plate has been endowed with electricity. Now touch it with a dry glass rod or a rod of vulcanite; it will still maintain its electrical property. Now touch it with the finger, and it will be found to have lost its electrical property.

(*c*) The metal plate can be charged with electricity by placing it on a rubbed plate of vulcanite, and lifting it up with a glass or vulcanite handle. Charge a plate in this way, and place it on a glass support. Now take a brass ball suspended by a dry silk fibre, and fix it in a position near to the plate. Take a wire long enough to join the ball to the plate and hold it by a vulcanite handle. Now connect the ball to the plate by means of the copper wire. The ball will be found to be electrified.

These experiments show the difference there is between such bodies as copper and such bodies as glass in their capacity for maintaining the electrical property. If an electrified body is connected to an unelectrified body by a copper wire, both bodies are found to be endowed with electricity. The copper wire has allowed the electrical property to be given to the second body; the electricity is said to have been conducted along the copper wire: a dry glass rod would not allow of the electrification passing from one body to the other. Copper is said to be a good conductor, and glass a bad conductor of electricity.

Substances may be arranged in order of their quality as regards conducting electricity; and, generally speaking, they will be capable of being arranged into two groups,—one set of substances which conduct well and are called conductors, another set which conduct badly and are called non-conductors or insulators.

These experiments also show how a conducting body—such as a brass ball suspended by a silk thread—can be charged with electricity by conduction.

Conductors.	Non-conductors.
Metals	Caoutchouc, ebonite, shellac
Acids	Porcelain
Saline solutions	Air and dry gases
Water	Dry paper
Flame	Silk
Water vapour	Mica
Linen	Glass
Cotton	Essence of turpentine

Since water and water-vapour are conductors, it is necessary that all non-conductors should be well dried and kept dried. Glass has the property of allowing water to condense on its surface rather freely, hence great care is required to keep it dry. A coating of shellac is often used.

If it is required to charge a conductor—the conductor must be supported by non-conducting bodies—*i.e.* on glass stem, or suspended by silk—it is then said to be insulated. If it is connected to the earth—which is a great conducting body—it is uninsulated.

Non-conductors are also called "insulators," and for a reason to be explained later they are called dielectrics.

Electrostatic Induction. EXPT. 3.—Place a metal vessel on a sheet of glass (coated with shellac). Suspend a small brass ball by a silk thread, and charge it by conduction as in Expt. 2 (*c*). Now suspend the ball within the vessel. It will be found that the vessel is electrified, for it will attract shreds of silk, paper, etc. Take the ball out without touching the vessel, and the vessel will exhibit no signs of electrification. The vessel is only electrified whilst under the influence of the charged body. It is said to be electrified by induction. Suspend the charged body within the vessel as before, and whilst it is within, touch the vessel, so that it is connected to the earth. Remove the finger and then remove the charged body from the vessel, taking care that it does not touch the vessel. The vessel will now be found to be electrified. This is the method of charging a body by *induction.* The charge may be proved to be of opposite kind to that of the charging body, and is said to be *induced.* But before proceeding further in investigating this and other laws of statical electricity, we must consider.

(i) An instrument for more easily obtaining a charge of electricity;

(ii) A more delicate and reliable instrument for detecting the presence of electrification.

i. The **Electrophorus.**—This is an instrument founded on the result of Expt. 2 (*c*) (see Fig. 74). It consists of a disc of insulating material, vulcanite or paraffin, fixed on a metal base called the sole. There is a smaller metal disc furnished with a glass handle and generally with a brass knob. Rub the vulcanite lightly with fur, and place the disc on it, and touch the disc. Now remove the disc. It will be found charged by induction from the vulcanite with the opposite kind of electricity, and a spark can be obtained. The disc may then be replaced on the vulcanite, touched, and withdrawn, when it will be found again charged; and this process can be continued until the charge on the vulcanite has leaked away. This instrument is called an electrophorus (electricity-bearer), and its exact action will be explained on page 174.

ii. The **Gold Leaf Electroscope.**—Take a flask and fit it with an india-rubber stopper. Through this pass a short ebonite rod, perforated to allow of a brass rod passing through. This rod is fitted with a brass disc at the top, and with two gold leaves at the bottom. The flask must be well cleaned and dried before putting the cork into it (see Fig. 57).

If now the instrument is charged by means of the electrophorus, the leaves will repel each other, and will remain at the same distance apart if there is no leakage by conduction.

In order to keep the instrument dry, strong sulphuric acid may be placed in the flask.

To increase the sensitiveness of the instrument, and for other reasons which will be understood later, it is advisable to surround the instrument as much as possible with conducting material,—coating the outside with tinfoil is an easy way of doing this (see Fig. 58).

EXPT. 4 (*a*).—*To charge the electroscope by induction.* Charge a glass rod and bring it near to the disc of the instrument, then touch the disc for a moment and afterwards withdraw the rod. Whilst the rod is near it before touching, the leaves will diverge, showing that the instrument is charged. When the instrument is touched the leaves will collapse, for, the instrument now forming part of the earth, the charge on the leaves is less and the divergence less. After withdrawing the rod the instrument will be found charged.

EXPT. 4 (*b*).—*To investigate the nature of the charge of a body.* Charge the electroscope by induction. Charge a glass rod with positive electricity by rubbing it with dry flannel. Now bring the glass rod carefully up to the disc of the electroscope, and notice whether the leaves diverge or collapse. In the same manner bring up the charged body to the disc, and notice whether its effect is the same or the opposite of that produced when the glass rod is brought up.

In this way the nature of the charge obtained when various substances are rubbed together may be investigated, and it will always be found that one of the two substances is charged with positive and the other with negative electricity.

In the following list—which the student should verify—any substance becomes negatively charged when rubbed by one which precedes it.

+
Catskin
Polished glass
Paper
Flannel
Silk
Metal
Ebonite
Sealing-wax
Sulphur
Gutta-percha
Rough glass
−

EXERCISES

1. Two pith balls hang side by side by two damp cotton threads. State and explain what happens when an excited glass rod is brought gradually near the two balls from below.

2. Give reasons for assigning the terms *positive* and *negative* to the charges of electricity obtained by rubbing a glass rod with a silk rubber.

How may negative electricity be obtained from a metal rod when rubbed with a piece of flannel?

3. Given a glass rod and a flannel rubber, show how to charge an insulated brass ball (1) with positive, and (2) with negative electricity.

4. Describe the construction of a gold-leaf electroscope, giving reasons for the choice of material in each part.

A conductor is surrounded by a glass case, and fixed on an insulating support at some distance from the electroscope. It is not possible to touch the conductor, nor to bring it close to the electroscope. How would you ascertain if the conductor is charged, and of what sign the charge is?

5. Explain how to charge an electroscope with positive electricity by induction. Show how to find the nature of the charge of an insulated conductor.

6. Describe the construction and explain the action of the electrophorus. State exactly what you must do in order to get a succession of sparks from an electrophorus.

SECTION II

On the Electrostatic Measurement of Electricity

1. **Faraday's Ice-pail Electroscope.**—Before proceeding to the experiments which will justify our regarding electricity as a *quantity*, the following experiment due to Faraday will provide an instrument of greater precision.

Place a tin or copper vessel 6″ deep, and say 3″ diameter, upon a vulcanite or glass plate. Join this vessel by means of a fine wire to the disc of an electroscope.

(i) Now take a brass ball attached to a vulcanite rod, and charge it "by a few sparks" from the electrophorus. Place the ball well within the vessel without touching it. The leaves of the electroscope will be found to diverge, but the deflection of the leaves will always be the same wherever the ball may be—provided it be well within the vessel. Now let the ball touch the vessel, and it will be found that the divergence of the leaves will be unaltered. Upon withdrawing the ball it will be found to have lost its charge.

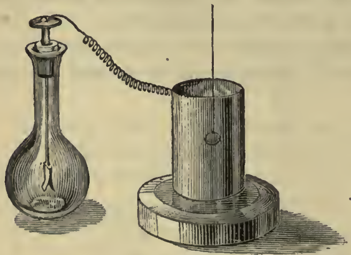


Fig. 57.

(ii) Take a small cylindrical metal vessel 3″ by 2″ diameter, to which a vulcanite handle is attached. Run some paraffin-wax into the bottom of the larger tin vessel, to form an insulating sheet about $\frac{1}{4}''$ thick. Charge a ball and place it within the larger metal vessel which is connected up with the electroscope, and observe the deflection of the gold leaves.

For this purpose it is better to use an electroscope with a graduated scale behind the leaves, as shown in Fig. 58. Now place the ball within the smaller metal vessel and discharge it into the vessel, and then place the vessel within the larger one and observe the deflection of the leaves. This deflection will be found the same as before.

Hence the divergence of the leaves produced when a charged body is placed within a given vessel is independent of the nature of the charged body, or of its position within the vessel (provided it is well within the vessel). We may say, therefore, that it depends only upon the charge of the body, and we may define equality of charge in the following way:—

Two bodies are said to be equally charged—or to have equal charges of electricity—if they produce equal deflection of the leaves of an electroscope when placed within a hollow vessel connected to the electroscope.

These experiments were performed by Faraday with ice-pails of different sizes, and the experiment is sometimes known as Faraday's ice-pail experiment. The apparatus will be frequently used, and may be referred to as Faraday's ice-pail or cage electroscope. Instead of placing the metal vessel upon a stand and connecting it to the electroscope, it may be placed directly on the disc of the electroscope.

It may be well to observe at once that a conductor can be charged with the entire charge of another conductor, if this charged conductor can be placed within it so as to touch it internally.

2. Electrostatic Measure of Electricity.

EXPT. 1.—*To charge a given hollow conductor with any number of times the charge of a given body.*

Set up the ice-pail electroscope, and place the vessel to be charged on an insulating stand. Charge a ball by means of an electrophorus, and observe the deflection of the leaves when placed within the pail of Faraday's electroscope. Take the ball out and discharge it into the given hollow conductor. Charge the ball again, and again observe the deflection of the leaves. If the ball has too great a charge touch it with a large insulated conductor so as to take away a small portion of the charge. If the charge is too small it must be further charged by the electrophorus. Having adjusted the charge so that it is the same at first, the ball can be discharged into the hollow conductor.

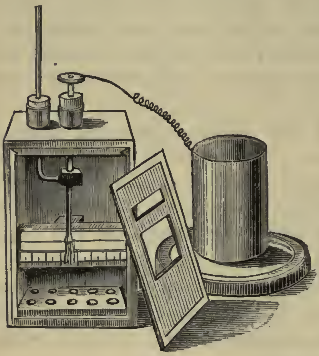


Fig. 58.

In this way the vessel can be charged with any number of times the charge of a given body.

Expt. 2.—*To charge a body with one-third the charge of a given body.*

Take three spheres of equal radius and attach thin vulcanite handles to them. Charge one of the spheres from the electrophorus. Now place the spheres in contact symmetrically, so that the charge divides amongst them. Test the charges of each ball separately. They will be found equal; so that each ball is now charged with one-third the charge of the first ball.

A Faraday's ice-pail electroscope could now be calibrated to measure quantities of electricity. All that would be necessary would be to observe the deflection of the leaves when a body charged with a certain charge was placed in the pail. Then charge a body with 2, 3, 4, etc. times this charge, and observe and mark the deflection. In this way we have a method of measuring charge.

All that is necessary is to adopt some given charge as unit of quantity. That charge which in a given ice-pail electroscope will produce a given deflection may provisionally be taken as the unit. Such a unit would soon be found unsuitable, as it would be difficult to reproduce with accuracy, and depends too much upon the slightest changes in the electroscope. We will see later that for many important purposes the quantity of electricity adopted as unit in Chapter II. can be taken as unit; but for other purposes, chiefly theoretical, it is necessary to take a unit based on absolute dynamical effects produced by direct electrostatic action.

3. Some Quantitative Electrostatic Experiments.

EXPT. 1.—*To prove that equal quantities of positive and negative electricity are created by friction or contact.*

Set up the pail electroscope. Attach the flannel rubber to a glass or ebonite insulating rod. (This is necessary because the flannel will conduct the charge through the hand.) Rub a glass rod with flannel, and place this rod within the pail, and notice the deflection. Now take out the rod and insert the flannel. The deflection will be the same, showing that the charges are equal. Now place the rod and flannel in the pail together. The deflection will be zero—showing that the charges are of opposite kinds.

EXPT. 2.—*To prove that when electricity is induced on a conductor, equal quantities of positive and negative are induced.*

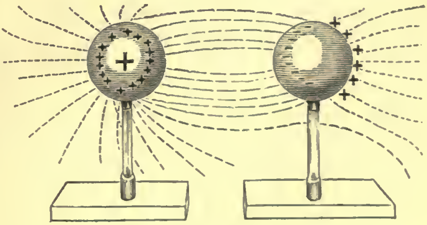


Fig. 59.

Take two brass balls fixed by vulcanite rods to wooden bases, so as to be well insulated.

Place them at a short distance apart, and connect them by a fine wire.

Now charge the disc of an electrophorus with positive electricity, and bring it up near to one of the balls A as in the figure. Then take away the wire by an insulating handle so as to insulate the balls from each other. Test the charge of each ball in the pail. They will be found equally charged.

Place them both within the pail. There will be no deflection—showing that the charges are equal and opposite.

4. **On Density of Electrification.**—If we take a small pliable thin disc to which an insulating handle is attached,

and place it so that it touches a charged conductor,—the disc will practically form a part of the external surface of the conductor, and will be charged with the electricity which was originally on the part of the surface it touches. If this disc be taken away at right angles to the surface, no electricity will pass from it to the conductor. The disc will therefore be charged with the quantity of electricity which was originally on the area of the conductor touched by it. This quantity can be measured by a pail-electroscope. In this way the quantities of electricity on each equal area of the conductor can be compared.

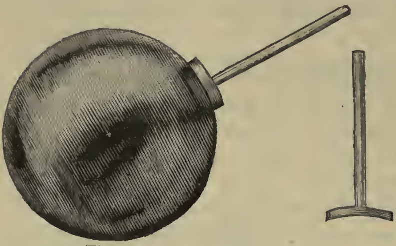


Fig. 60.

The quantity of electricity per square cm. of the surface at a given point is called the **surface density** at that point.

This disc with the insulating handle is called a proof-plane.

A long series of experiments was performed by Coulomb in order to compare the densities at different points of charged bodies.

The following diagrams will give approximately some idea of the electrification of conductors of different shapes.

(i) Fig. 61 shows the electrification of a sphere placed in a large room. The density is uniform.

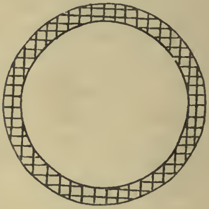


Fig. 61.

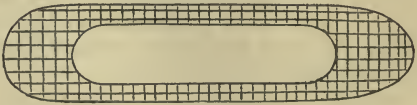


Fig. 62.

(ii) Fig. 62 shows the electrification of a cylinder with hemispherical ends—charged and placed in a large room.

(iii) Fig. 63 shows the electrification of a conical-shaped conductor, charged and placed in a large room.

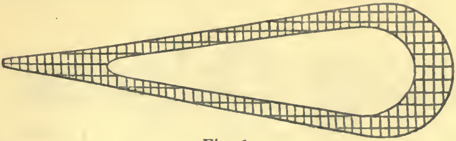


Fig. 63.

(iv) Fig. 64 shows the electrification of a cylinder when electrified by induction from a large sphere charged with positive electricity. From *ab* towards the sphere it is charged with negative electricity—and from *ab* to the right, away from the sphere, it is charged with positive electricity.

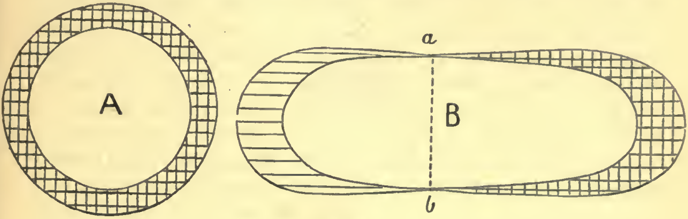


Fig. 64.

EXERCISES

1. Describe Faraday's ice-pail experiment, and show clearly what it proves: and how it gives a means of defining equality of charges. How is it that a simple electroscope is not able to test equality of charges?

2. Describe the experiments to prove that electricity is a measurable quantity. Show how to calibrate a cage electrometer for charge.

3. Why is it convenient for electrostatic conductors to be made hollow? Show how to charge a hollow conductor successively with equal charges.

4. A sphere is charged, and found to possess too great a charge. Given a small insulated body, how would you proceed to take (i) a small quantity; (ii) an exceedingly small quantity; (iii) a large quantity away from the sphere?

5. In Expt. 2. what is the nature of the charge on D (i) when it is insulated; (ii) when connected to the earth.

6. Define surface density at a point of an electrified body and describe how to show that the density at the ends of a long cylinder with spherical ends is more than twice the density at the middle.

7. Trace the changes in the distribution of electricity on an insulated

cylinder as a large charged sphere is brought up to it in the direction of the axis of the cylinder. Illustrate your answer by three drawings.

SECTION III

Fundamental Experiment on which is based the Law of Force between two Quantities of Electricity

1. Wherever a quantity of electricity is found, an equal quantity of electricity of the opposite sign will be found somewhere. Faraday made elaborate researches in order, if possible, to discover a body charged with electricity absolutely, but he found that there was always an equal quantity of the opposite kind. If electricity is created by friction an equal quantity of both kinds is created. If a body be charged with positive electricity and be placed in a large room, and all conducting bodies removed, an equal quantity of electricity of the opposite sign is distributed over the walls of the room. This tends to confirm the theory that electrification is only the manifestation of the strain of a medium which occupies the space of the insulators. Whilst engaged in these researches Faraday discovered that there was no charge on the inside surface of a hollow conductor provided there was no charged body inside: and that there was no force at a point inside a closed charged conductor. To prove that there is no force inside a closed charged conductor, Faraday constructed an iron box, into which he went and took every instrument for detecting the presence of electricity or electrical force; but although the box was well insulated and was kept highly charged by means of a Holtz machine so that long sparks could be drawn from the outside — there was no sign of electrification inside. In Faraday's own words, "I put a delicate gold-leaf electrometer within the cube, and then charged the whole by an outside communication, very strongly for some time together; but neither during the charge or after the discharge did the electrometer or air within show the least signs of electricity. . . . I went into the cube and lived in it, and using all other tests of electrical states, I could not find the least influence upon them, though all the time the outside of the cube was powerfully charged, and large sparks and brushes were darting off from every point of its outer surface."

That the electricity resides on the outside of a hollow conductor had been proved by Biot. (i) Biot's apparatus was the following: A brass sphere A was placed on an insulating stand, and two hollow hemispheres,

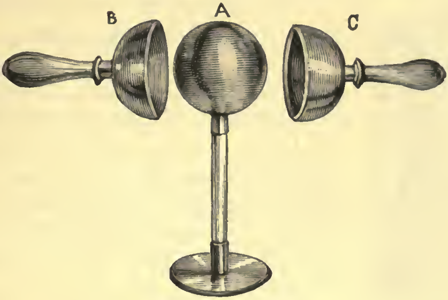


Fig. 65.

B, C, furnished with insulating handles, were formed so as just to cover the sphere. The sphere A was highly charged, and then the uncharged hemispheres were placed on it. On removing them it was found that all the charge of A had gone to B and C. (ii) Coulomb's experiment was to highly charge an insulated hollow sphere, and test the charge on the inside surface by means of a small proof plane. (iii) Faraday's experiment was the following. ABC is a muslin conical bag fitted to a ring AB, CDE is a string by means of which the bag can be completely turned inside out. The bag, being insulated, was charged and then tested by the proof plane. The electricity was found on the outside. The bag was then turned inside out, and again tested, when it was found that all the electricity had gone to the outside.

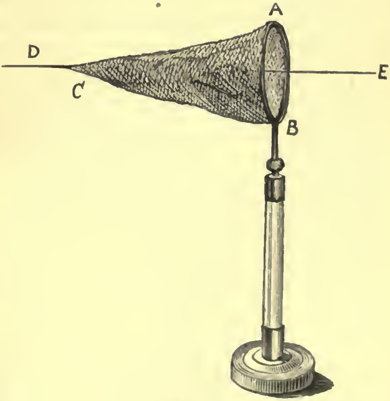


Fig. 66.

The mathematical theory will prove that if there is no force inside a hollow conductor, then there can be no charge on the inside: so that the whole rests on the experiment to prove that there is no force inside, and this can be put to the most rigid test.

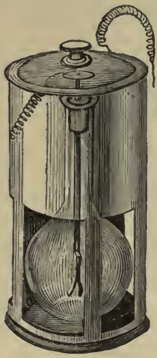


Fig. 67.

The student may perform these experiments in the following manner:—

Expt. 1.—*To prove that there is no force inside a closed charged conductor, provided there is no charged body within.*

Make a cage of wire netting or gauze, into which the electroscope can be placed. Place it on an insulating sheet of glass: and join the inside of it to the electroscope.

Now charge an electrophorus and charge the cage.

It will be found that even when sparks can be drawn from the outside, no effect is produced on the leaves of the electroscope.

Expt. 2.—*To prove that there is no charge on the inside of a closed charged conductor, provided that there is no charged body within it.*

Take a hollow sphere or cylinder on an insulating stand and charge it. Now apply a proof plane to the inside of the vessel, and then test the proof plane for charge by putting it into the vessel of the Faraday electroscope. It will be found to have no charge.

This shows that there is no charge inside a hollow charged conductor.

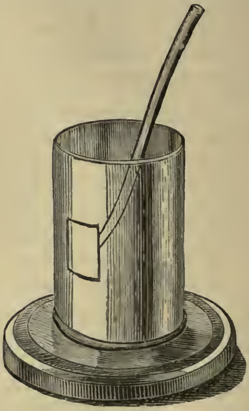


Fig. 68.

2. **On the Laws of Electric Force.**—The force between two small spheres charged with electricity has been proved by Coulomb and others to obey the following laws:—

Law I.—To vary directly as the quantities of electricity on the spheres.

LAW II.—To vary inversely as the square of the distance between the two bodies.

LAW III.—To vary inversely as the specific inductive capacity of the dielectric.

The electrostatic unit of quantity of electricity may be taken to be that quantity which, placed at a distance of one centimetre from an equal quantity—the dielectric being air—will repel it with a force of one dyne.

This quantity is based on a statical measure of the electric effect, and is therefore called the **electrostatic** unit. In Chapter II. we have taken a unit of quantity of electricity based on the magnetic effect of a current of electricity—based, as we believe, on the effect due to the motion of the electrostatic tubes. This quantity is called the electromagnetic unit. The experiments the student has performed will lead him to believe that the electromagnetic unit is a very much larger quantity of electricity than the electrostatic unit. If he considers how easy it is to charge two small balls which, placed at a distance of 1 cm. apart, will repel each other with a greater force than one dyne, and if he then considers what mass of copper would be deposited in a copper sulphate voltameter on discharging these balls through the voltameter, he will readily believe that the E.M.U. is much greater than the E.S.U. In fact, experiments show that there is every reason to believe that the E.M.U. contains the E.S.U. about 30,000 million times, and further that this 30,000 million gives in cm.-per-sec. the velocity of light. If a small sphere be charged with the electrostatic unit, and be rotated in a circle of 1 centimetre radius at the rate of 30,000 million times per sec., then in one sec. we may say that 30,000 million E.S.U. has passed round the circuit, *i.e.* in one second the E.M.U. has passed, *i.e.* the current will be one E.M.U. of current. Theory points to the fact that this would give rise to a magnetic field, and that the intensity at the centre of the circle would be one dyne per unit. This theory has been directly confirmed by experiment.

From these laws the force between two quantities, q, q'

E.S.U. placed at a distance of r cm. apart in a dielectric of specific inductive capacity K is given in dynes by the formula

$$F = \frac{q\,q'}{Kr^2}.$$

Until the student has studied the section treating of specific inductive capacity, he may consider the actions to take place in air where $K = 1$. The law of inverse squares originally rested on experiments performed by Coulomb, using for the purpose his Electric Torsion Balance—an instrument similar to the magnetic Torsion Balance of Chapter I.; the difference being, that instead of the magnet a light rod having two gilt-covered pith balls at each end was suspended by a silk fibre. One of these balls was charged, and then a second charged ball brought up to it, and the law of inverse squares investigated as in the corresponding problem in magnetism.

It has, however, been proved that if there is no force at any point inside a closed charged conductor, then the law of force must be that of the inverse squares. The experiment given above is then the best evidence of the truth of the law.

EXERCISES

1. Describe an experiment which shows that there is no electricity on the inner surface of a closed charged conductor. What important inference respecting the law of electric action is drawn from the result of this experiment?

2. What do you understand by an absolute charge of electricity? One vessel is placed within another and is insulated from it. The inner vessel is charged. How would you prove that the outer vessel has an equal and opposite charge on its inner surface? Is there any charge on the outer surface, and if so what is the charge, and where is the equal and opposite charge?

3. An electroscope is placed within a closed metal vessel, and an excited glass rod is brought up to it, what will happen to the leaves? Water is slowly poured into the vessel until it reaches the bottom of the cylinder, the glass rod remaining in position, state and explain what happens.

4. How would you show experimentally that when positive electricity is generated by friction an equal quantity of negative electricity is also generated? Explain the advantage of using nearly closed hollow conductors in the study of electrostatic phenomena.

5. Upon what kind of experimental evidence does the law of inverse squares depend? Describe as fully as you can the experiments which have been made to verify the law.

6. What is the electrostatic unit of electricity? What reasons have we for believing that it is much smaller than the electromagnetic unit?

7. It is required to compare a standard of length in one country with the standard of length in another, without removing these standards from their country. How can this be done?

SECTION IV

On the Electric Field

1. The part of space in which electric phenomena may be observed is called an "Electric Field." The force which would be exerted upon a small body charged with unit quantity of electricity, if it were placed at a given point, is called the **Electric Intensity** at that point of the field. It is measured in dynes per unit quantity of electricity. For the complete determination of the Electric intensity at a point of a field of force, the following quantities must be known :—

(i) The direction of the intensity.
(ii) The sense.
(iii) The magnitude.

(i) The direction of the intensity at a point may be determined experimentally by taking a short piece of cotton thread, and suspending it at the point by a long silk fibre. The conducting cotton will be electrified by induction and will set in the direction of the intensity at the point. This is a method similar to the method of tracing the direction of the magnetic intensity at points of a magnetic field by means of iron filings.

If a line be drawn in the field such that the tangent at any point of it is the direction of the intensity at that point, this line forms a line of force. The path in which a light pith ball will move is a line of force, if the weight of the ball can be neglected compared with the forces acting upon it due to the electric field.

The determination by experimental means of the nature of the distribution of the lines of force in an electric field presents difficulties much greater than are found in the corresponding problem in magnetism. The direction of the lines can be inferred in some special cases by the laws of symmetry, in others they can be deduced by analogy with magnetic fields; and in a few cases they can be discovered by mathematical methods of a more or less complex nature.

Faraday was the first to turn the attention from the conducting bodies to the medium surrounding them, and by a long series of experiments he was led to seek the explanation of the observed phenomena in the action of this medium, and to formulate a condition of the medium which would lead to a natural explanation. When a body is charged with electricity we may regard the medium surrounding the body as having been put into a condition of stress. Strictly speaking, it is the medium which is charged, and it is the medium in which the energy resides. It is the essential characteristic of a non-conducting substance that it can maintain the condition of electric stress — and of conductors that they cannot maintain the stress. Non-conductors, looked at from this point of view, are called dielectrics, because it is through them that the electrical actions are propagated. As in the case of the magnetic field, Faraday proceeded to map out the dielectric field by means of lines of force, and to consider the dielectric as made up of tubes of force in a condition of stress. The properties of these tubes are similar to the properties of the magnetic tubes. We are to regard them as being in a state of tension along lines of force, and as exerting lateral pressure against each other at right angles to lines of force. An electric tube of force begins on one conductor and ends on another; it does not return to the same conductor, nor turn round upon itself. Faraday spent a long time in establishing this fact. Wherever there was a quantity of positive electricity Faraday found there existed, connected with it, an equal quantity of negative electricity somewhere. Further, the lines of force, as can be easily verified with the cotton thread, run down into a conductor *at right angles* to the conductor. We may therefore regard conducting bodies, which are not capable of maintaining electric stress, as being placed in a non-conducting dielectric capable of maintaining stress, and the tubes of force (or tubes of induction) indicate the nature of the stress, and run from one conductor to another. One end of the tube abutting on one conductor gives that conductor the appearance of a positive charge, and the other end will run down on to another conductor and will give that part of the conductor the appearance of a negative charge.

We will now attempt to give explanations of some of the phenomena described in the previous sections according to Faraday's tubes of force.

The following figures give the distribution of the lines of force in a few cases :—

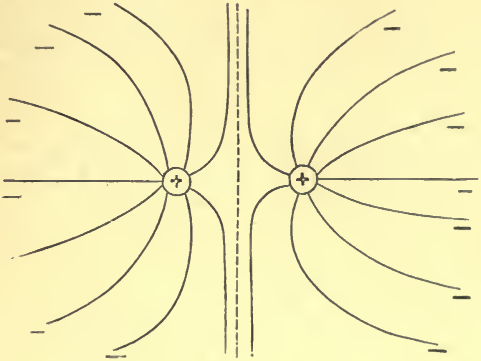


Fig. 69.

(i) Fig. 69.—Two positively-charged spheres placed in a large room. The lines of force are as in the figure, their positive ends abut on the charged bodies, and the other ends abut on the walls of the room, or on any conducting bodies placed in the room.

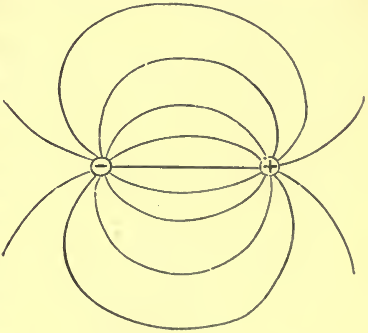


Fig. 70.

As in the case of two north poles, we see that there are more tubes pulling each sphere to the side of the room away from the other sphere. Hence the tension of the tubes joining the balls to the sides of the room draw the balls further apart, and they appear to repel each other.

(ii) Fig. 70.—Two charged balls, one positive the other negative, placed a distance apart in a large room. The figure shows that the tension of the tubes will draw the balls closer together.

(iii) Fig. 71 shows a positively-charged body A placed near

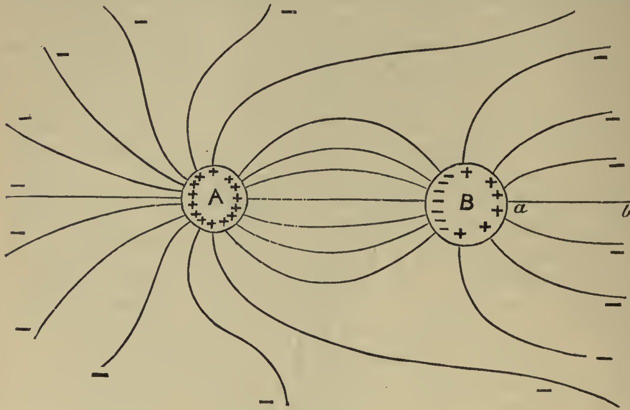


Fig. 71.

an originally uncharged body B—the whole being in a large room.

The charge of A may be regarded as 18 units, denoted by the 18 lines. Before B was placed in position the lines from A would run straight out to the walls of the room.

On bringing B into the dielectric, the lines which existed in the space now occupied by B were destroyed, with the result that the lateral pressure against the adjacent tubes would be removed, and the lines or tubes would bend in as in the figure —as many lines running out from B as run into it, viz. 7.

It is clear that the nearer B be brought to A the more lines will be destroyed and the more will run down into itself. The effect of B is to break certain lines—in this case 7.

B is said to be charged by induction; in fact its charge is entirely due to the existence of the lines already in the field; and we see that it is charged with equal quantities of positive and negative electricity.

Now suppose B is joined to the walls of the room by a metallic wire, along *ab* say. This wire will destroy the tube along which it passes, and hence the adjacent tubes will be relieved of the lateral pressure and their ends will run along the wire, so that the tube shrinks to molecular dimensions, and heat is created. As the ends of all the seven tubes leading from B to the room are metallically joined, all these tubes will shrink and disappear, and the remaining tubes from A will bend round as shown in Fig. 72, the ends abutting on B, moving round to the other side of B as shown.

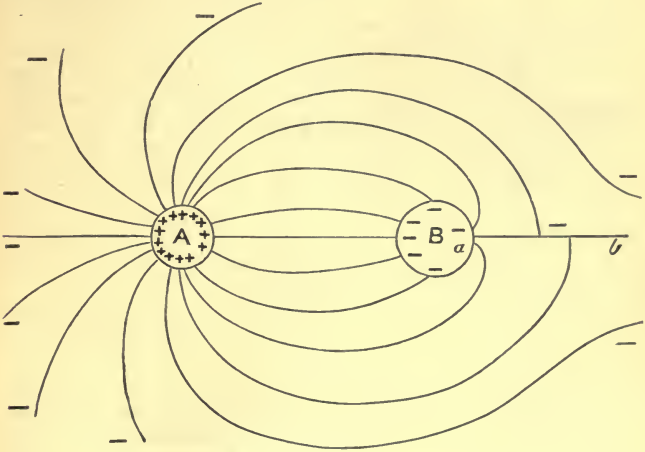


Fig. 72.

B is now charged with negative electricity only, and can be removed without losing its charge.

(iv) It follows from the properties of the tubes of induction that there can be no tubes inside a hollowed conductor, unless there are other conductors inside the vessel, and insulated from it. For if there are no other conductors inside the hollow conducting vessel, the tubes will begin and end on the same conductor, and this is impossible. Hence there are no lines of force within such a hollow conductor, and therefore the intensity at every point within the conductor is zero.

Again, if a charged body such as the sphere in Fig. 72 be introduced within a closed vessel the lines of force from this conductor will run down on to the inner surface of the vessel. Hence the negative charge on the inner surface of the vessel will be equal to the positive charge of the sphere.

(v) Fig. 73 gives the distribution of the lines of force when

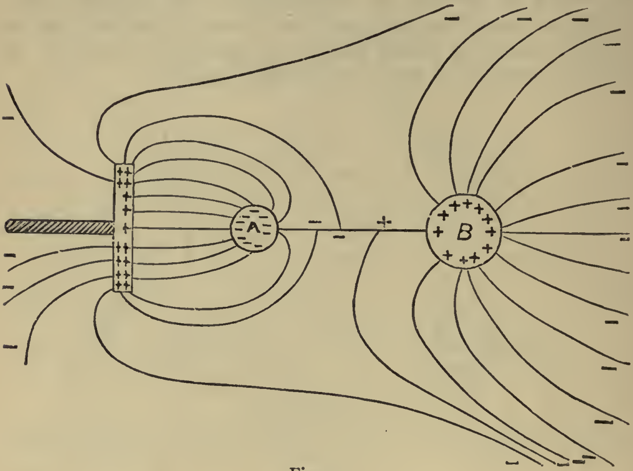


Fig. 73.

the two balls A and B are under induction from the positively-charged electrophorus disc in the experiment 2 (page 161) above.

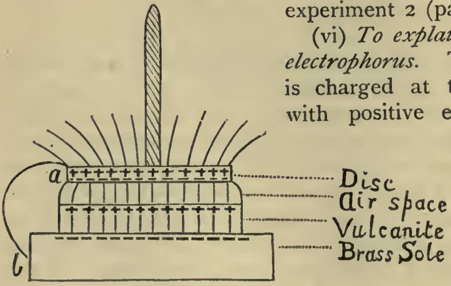


Fig. 74.

(vi) *To explain the action of the electrophorus.* The vulcanite disc is charged at the upper surface with positive electricity. Hence lines of induction pass from the surface upwards through the air between the vulcanite and the disc, and downwards through the vulcanite to the brass sole. Hence the brass sole is charged with negative electricity on its upper surface, whilst the disc is charged with negative electricity on its lower surface. From the top of the disc lines start and run towards the earth. In fact the disc is charged with positive and negative electricity by induction. Now join the disc and the sole either by the hand or by a wire, and both to the earth through the body. This will cut through the lines running from the upper part of the disc, and from the lower part of the sole, and the disc will be charged with positive electricity.

Before connecting the disc to the sole, it will be seen that as many lines run out of it towards the earth (or walls of the room) as ran into it from the vulcanite plate, hence no work will be required to lift the disc. When, however, the lines connecting the earth and the disc have been cut through and destroyed, the disc is only acted upon by the lines from the vulcanite, and work will have to be done in pulling the two apart.

Hence the energy of the charge of the disc is created by the work done in separating it from the vulcanite.

The direction of the lines of force in some special cases of importance can be inferred from the symmetrical arrangement of the conductors and dielectric.

(1) In the case of two parallel plates separated by a uniform dielectric the lines of force are parallel to each other and perpendicular to the plates.

(2) In the case of a sphere placed within a hollow concentric spherical shell, and separated from it by an insulating medium, the lines of force radiate from the centre, just as if there were a charge at the centre.

(3) In the case of a cylinder placed within a hollow co-axial cylindrical shell, and separated from it by a uniform dielectric, the lines of force radiate at right angles from the axis.

(ii) The **sense** of the intensity at a point is the "way" the force acts upon the *north* pole placed at the point. For example, if the field is due to a positively-charged body placed at O, then the sense of the intensity at a point P, will be along OP from O towards P.

If AB is a line of force, and if at any point P, the force on unit quantity of electricity is from A towards B, then the sense of the intensity is denoted by the arrow in the figure, and is from A towards B.

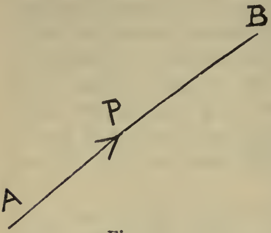


Fig. 75.

(iii) The following is an experimental method of comparing the magnitude of the intensities at points of a field.

Take two similar and equal thin metal discs, attached to insulating handles, and place them together, so that their planes are perpendicular to the direction of the intensity. If the direction of the intensity is from B to A, then positive electricity will be displaced in the direction of the intensity, and A will be charged with positive electricity. Negative electricity will be attracted in the direction AB, and will be displaced on to B, and B will be negatively electrified. Now separate them, and measure the charge of each in the cage electrometer. The charges will be equal but of opposite sign. Now the charges induced on A and B have been proved by Coulomb to be directly proportional to the intensity at a point on A or B. Hence by comparing the charges induced on the same disc at different points of the field, the intensities at these points can be compared.

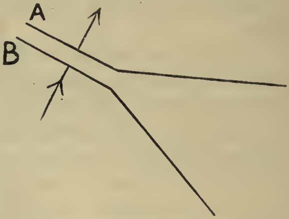


Fig. 76.

If the discs were placed parallel to the lines of force, and then separated, they would be found to possess no charge; for the electric displacement takes place only in the direction of the electric intensity at the point.

EXERCISES

1. What do you understand by an "Electric Field"? Why "field"? What is *intensity?* How does it differ from force? Give analogies.

2. What must we know about a force to completely determine it? What about intensity?

3. How can an electric field be plotted out by "lines of force"? What is a line of force? Give analogies.

4. How can a body be discharged? What is the condition of the medium during the discharge? How can you test these effects?

5. A sphere is placed within a large oblong room, the walls of which are conductors. Draw the lines of force when the sphere is charged with positive electricity. If a second insulated but originally uncharged sphere be brought into the room up to the sphere but not touching it, what changes are produced? Draw the lines of force. Now let the two spheres touch, what happens? Draw the lines.

6. Why do two positively charged small spheres repel each other? If one were large and the other small, would they necessarily repel each other?

7. Explain by means of the lines of force how a body is charged by induction.

8. Explain the action of the electrophorus. Where does the energy "of the spark" come from?

SECTION V

On the Condition necessary that Electricity shall pass from one Body to another. Potential.

1. Place a small brass ball on a vulcanite stand, and then charge an electrophorus. Now charge the ball by touching it with the electrophorus. Test the charge of the ball and of the electrophorus by the cage electroscope. The disc will still be found to have a considerable charge and the small ball a small charge; but it will be found impossible to give any of the charge of the disc to the ball in the same manner, and under the same conditions as before.

Now place a large insulated sphere close up to the small ball, but not touching it, and then touch the small ball with the disc as before, and test the charge of the small ball. It will be found to have increased; that is, the presence of the large sphere has made it possible for more electricity to pass from the disc to the small ball. If the sphere is joined by a wire to the gas or water pipes and thereby to the large conductor—

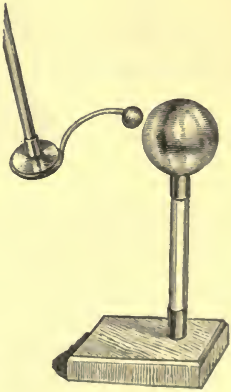


Fig. 77.

the earth, still more electricity will be found to pass to the small ball; and still more will be found to pass to it, if a sheet of glass or vulcanite or shellac be placed between the two spheres. If the sphere were removed and a contact made with the disc, it would be found that the small ball would give up electricity to the disc. Whether electricity will pass from one body to another does not therefore depend only upon the charge, but upon other conditions, upon the relation of other bodies to it, and the nature of the insulating medium. When electricity will not pass from one body to another they are said to be at the same *potential*. If positive electricity passes from one body A to another body B, A is said to be at a higher potential than B. The following apparatus will enable this property of electrified bodies to be more closely studied :—

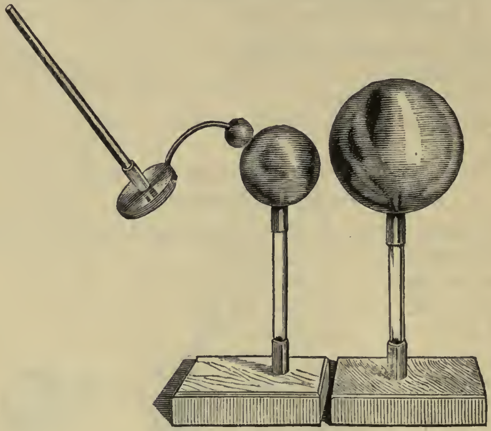


Fig. 78.

An insulated plate A is fixed on a slide stand, and a second insulated plate B is placed parallel to it, and so that it can be moved parallel to itself along the stand. C and D are similar plates at a considerable distance off. B is joined by a fine wire to an electroscope E. Charge B when A is present. Notice the repulsion of the leaves showing that electricity has passed into E. E and B are now at the same potential. Now draw A further from B, the leaves will diverge still further, showing that electricity has again passed from B to E, and therefore the potential of B is increased. Slide A nearer to B and the leaves will collapse, showing that as electricity has passed from E to B the potential of B has decreased. Try the effect of placing a sheet of glass between A and B, it will be found that the effect is the same as that when A is placed nearer to B, *i.e.* the leaves collapse, and therefore the potential of B decreases. Now join A to the earth, and perform the same experiments, the effects will be the same, but more distinct.

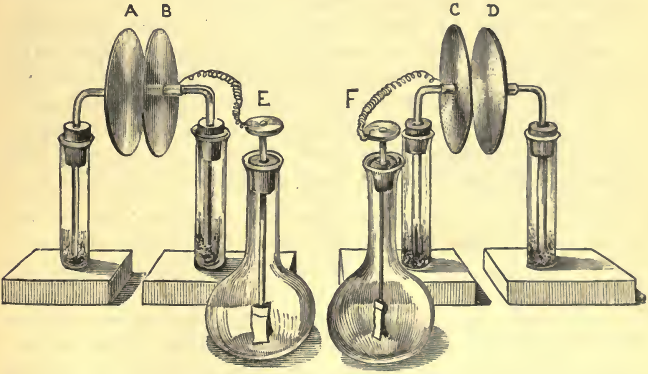


Fig. 79.

Again join C to a second electroscope F, and connect D to the earth, and charge C, the leaves in F diverging. Now join B and C by a fine wire. If there is no change in the deflection of E, there will be no change in the deflection of F, and consequently no interchange of electricity, and therefore C and B will be at the same potential. If the leaves of E diverge, those of F will collapse, showing that electricity has passed from C to B, and that the potential of C had been higher than that of B.

When B and C have been joined their potentials are equal, and since A and D are joined to the earth their potentials are equal. Now draw D further from C, the leaves in F will diverge, showing that the potential of C has increased, and is therefore greater than that of B. Now join CB, then as is to be expected the leaves in E diverge and those in F collapse, showing that electricity has passed from C to B, from a high potential to a low potential.

The converse effect would be produced if the plate D were brought nearer to C.

These experiments prove the statement made above. Hence we can give the following definitions:—

The potential of a conductor is its electrical condition in virtue of which it tends to give or receive electricity to or from other conductors.

Two bodies are said to be at the same potential if, when joined by a conductor, they neither give electricity to nor receive electricity from each other.

A body A is said to be a higher potential than B, if when they are joined by a conductor positive electricity passes from A to B, and be at a lower potential if positive electricity passes from B to A.

These definitions are analogous to the definitions of temperature, and the electroscope bears the same relation to the potential of bodies as the thermometer does to the temperature.

In order to measure difference of potential it is necessary to have an experimental method of determining whether two pairs of conductors have equal difference of potential: and to have a means of adding differences of potential.

In order to measure difference of potential we must have a more suitable instrument than the gold-leaf electroscope. Such an instrument was devised by Sir William Thomson (Lord Kelvin) and is called the **Quadrant Electrometer.**

Without giving the practical details of the instrument, the following are its essential features.

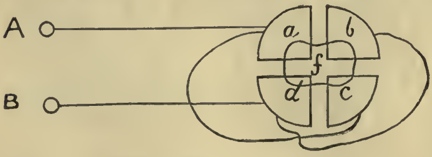


Fig. 80.

There are four quadrants of brass, each supported from a base by glass supports. The opposite quadrants are joined by fine wires. Above the quadrants, but close to them, is suspended a light aluminium fan *f* of the shape shown in the diagram. This is generally suspended by two cocoon fibres, and is provided with a mirror by means of which the deflection of the fan can be measured by the usual lamp and scale method. The fan is charged with electricity from an electrophorus, and when the opposite quadrants are at the same potential (by joining them) the fan is adjusted to set symmetrically between the quadrants. If now the terminals A and B be maintained at a difference of potential, there will be different charges on *a*, *c* and *b*, *d*. This will cause the fan to be deflected, and it can be proved mathematically that this deflection is proportional to the difference of potential of A and B.

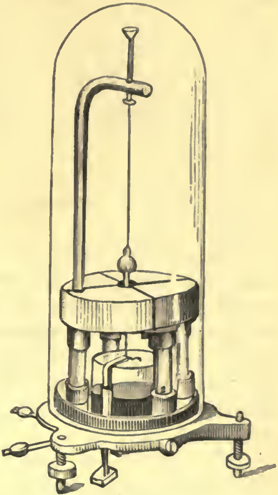


Fig. 81.

To maintain the fan's charge constant, a platinum wire hangs from it, and dips into sulphuric acid placed in a glass vessel, the outside of which is coated with tinfoil. The student will see later that this vessel, with its coatings, constitutes a Leyden jar, and that the charge on the fan will thereby be maintained constant, as any slight leakage from it would be replaced by the jar.

That the quadrant electrometer measures difference of potential may be verified in the following way:—Let A, B be two metal vessels, B being placed within A, but insulated from it by some insulating medium,—a glass bottle coated with tinfoil inside and out will do. Let C, D; E, F be two pairs of identically similar and equal vessels. Charge B, and connect A to D, and C to F by fine wires. Now join B to one pole of the quadrant, and A to the other, and take the reading. Then join D to one pole and C to the other; then F to one pole and E to the other. The deflections will be the same in each case.

Now join B to one pole and F to the other, the reading

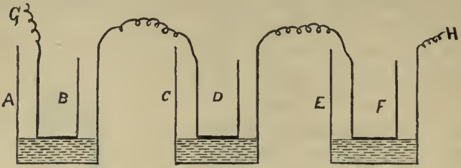


Fig. 82.

will give three times the difference of potential between A and B.

The quadrant electrometer can be made extremely sensitive, and can therefore be used to indicate small differences of potential. If the poles of a galvanic cell—say a Daniell cell—be joined to the two terminals of an electrometer, the electrometer will show a difference of potential. By using a number of Daniell cells and arranging them in series, multiples of this difference of potential can be obtained, and the instrument calibrated.

Now the difference of potential between the terminals of a Daniell cell is practically constant, and this difference may be taken as the unit difference of potential. It is very small compared with the difference of potential which is usually met with when the difference is caused by electrostatical means.

2. **On a Dynamical Measure of Difference of Potential.**—The following explanation will show the relation between E.M.F. as defined in Chapter II., and difference of Potential :—

Suppose the two insulated plates A and B are connected respectively to the two poles of a Daniell cell. The two plates will be charged. In fact the medium will be put into a state of electrostatic strain, and the whole space surrounding the cell can be mapped out by lines of force. If a key and a suitable galvanometer be introduced into the circuit it will be found that during the infinitely short time in which the plates are being charged, a current is flowing in the wire. This we would expect from the motion of the electrostatic tubes. The two plates are now raised to a difference of potential, and the cell can be disconnected. If the two plates be connected by a wire—the discharge will take place, and work will be done, and will appear in the form of heat, either in the connecting wire, or in the air if a spark passes between them. Now when electricity passes from one conductor to another work is done. If we cause the discharge to take place in instalments by means of a small discharging ball, we shall see that the work done when a given quantity of electricity passes across from one plate to the other depends upon the difference of potential. Suppose a very small brass sphere is suspended by a fine silk fibre and touches one of the plates. It will take off a charge by conduction, and will be drawn across to the other plate and discharge its charge. Work will be done by the forces due to the medium acting upon the ball; but when the discharge has taken place and the given quantity is passed across, the charges on the plates will be less, and therefore the forces acting upon the ball when charged with the same charge as before will be less and hence less work will be done. We see therefore that the work done when a small quantity of electricity is carried across from one plate to another, will depend upon the difference of potential between the plates, and may be taken as a measure of this difference.

Hence difference of potential may be dynamically measured by the work done when a unit quantity of electricity passes from one plate to the other. But this work done per unit quantity of electricity is precisely what has been defined as the E.M.F. between the two bodies, or parts of a conductor. Hence difference of potential is measured dynamically by the E.M.F.

If the two plates be connected by a wire so that a current of electricity passes, the quadrant electrometer will show that the energy of the cell maintains a constant difference of potential between the two plates, less, of course, than when no current is running, and greater the greater the resistance of the connecting wire.

We see therefore that the quadrant electrometer may be used to compare the E.M.F.'s of cells, and the E.M.F.'s between points of a current circuit.

The unit of difference of potential like the unit of E.M.F. will depend upon the unit of electricity adopted, and upon the unit of work.

If the unit of electricity is the electromagnetic unit defined in Chapter II., and the unit of work is the erg, then the unit of difference of potential is the electromagnetic unit of E.M.F.

If the unit of electricity is the Coloumb, and the unit of work the Joule, then the unit of difference of potential is the Volt.

Exercises

1. A metal disc is charged and insulated. It is required to transfer as large a charge as possible from it to an insulated brass ball, state the methods you would adopt.

2. Two metal discs A and B, equal in all respects, are placed in contact so as to form one disc, and are then insulated and charged. After separation their charges are equal. A metal disc joined to the earth is brought near to one of them, and then A and B are joined by a fine wire. Which will gain electricity, and how can it be proved experimentally?

3. An insulated sphere is charged. How would you experimentally determine whether its potential was above or below that of the earth?

4. Define difference of potential. What experiment must be performed to prove that P.D. is a measurable quantity? Does it follow that potential itself is measurable? What additional knowledge is required in order to measure potential? State some standards of P.D. which may be used.

5. When are two points said to be at the same level? A and B and C are three points in the same vertical. The difference of level A, C is twice that of A, B, will C be the same point in whatever manner difference of level is measured, *i.e.* by plumb-line and foot-rule or by principle of work? Define difference of level, and state a method of proving equality of difference of level: prove from your definition that the foot-rule plumb-line method is not correct.

6. A disc A is insulated and charged, and a disc B is placed parallel to it: it is then connected to the earth. Explain why the P.D. is thereby increased.

7. Give the essential characteristics of the quadrant electrometer.

8. Show how to use the quadrant electrometer to compare the E.M.F.'s of a Daniell and a Bunsen.

Section VI

Condensers—Capacity

1. In the experiment with the parallel plates A and B, in which A is connected to the earth, and B insulated, the potential of B decreases as the plates are brought nearer together. In order to bring its potential up to the original potential, it is necessary to give B a greater charge. Hence,

when the plates are nearer together, B will have a greater charge for the same potential, if the potential of A is unchanged. Again, let the pairs of plates A, B and C, D be identical in every respect, and separated the same distance apart, and let A and D be joined to the earth, and B and C joined. Charge B and C. Their charges will be equal and potentials equal. Disconnect B and C. Now draw D further from C, and then connect C and B. Electricity will flow from C to B, as will be observed by the deflection of the leaves. Hence B will now have a larger charge than C. This again shows that the quantity of electricity which B must be charged with so that its potential shall be equal to that of C, is greater than the charge on C.

In other words, these experiments prove that the charge a conductor has, when at a given potential, depends upon the relation of other conductors to it. By bringing two plate conductors near together, and connecting one to the earth, the other can receive a large charge under the given potential. Two conductors so arranged that a large charge can be given to one of them at a given potential form a **condenser**.

It can be proved mathematically, and can be verified experimentally, that the charge of a condenser is directly proportional to the difference of potentials of its two plates.

A direct experiment for testing this is the following:—Form a condenser of two tin vessels, one within the other, separated by a dielectric. Connect the outer vessel to the earth, and the inner one to one pole of an electrometer. Connect the remaining pole of the electrometer to the earth. Now charge a ball as highly as possible, and discharge it into the inner vessel, and read the difference of potential. Now charge the ball again with an equal charge, and discharge into the inner vessel, and read the deflection of the electrometer—and thus the difference of potential. It will be found that for a given condenser the charge is directly proportional to the difference of potential. For a given difference of potential (say a volt) a given condenser will have a definite charge—depending only on its geometrical dimensions and the nature of the dielectric. This is called the capacity of the condenser. It is somewhat analogous with the capacity of a gas-bottle, viz. the quantity of gas it will hold under standard pressure.

If C is the capacity of a condenser, then the condenser will be charged with C units of electricity when the difference of potential is unity. Hence the quantity of electricity it will be charged with when the difference of potential is E units is CE. If this is denoted by Q units, then

$$Q = CE.$$

If the unit of quantity is the coulomb, and the unit of difference of potential the volt, then the condenser which, when charged to a difference of Potential of 1 volt (*e.g.* by a Daniell cell) will have a charge of a coulomb, is said to have a capacity of 1 **Farad.**

If the student considers what takes place when a condenser is charged—how small the average current is, and how small is the quantity of electricity which passes in the exceedingly short time during which the current is passing, he will be prepared to accept from the results of mathematics that the capacity of a farad is very great. In fact it is so great that for practical purposes one-millionth of a farad is taken as a working unit, and is called a microfarad.

Since 1 farad is the capacity of a condenser which, when kept at a difference of potential of 1 volt (*i.e.* 10^8 abs. units), has a charge 1 coulomb, *i.e.* 10^{-1} abs. units.

$$\text{The farad} = \frac{Q}{E} = \frac{10^{-1}}{10^8} = 10^{-9} \text{ abs. units.}$$

Hence the farad is 1000-millionth part of the absolute electromagnetic unit.

Since the microfarad is the millionth part of the Farad it is

$$10^{-15} \text{ of E.M.U.}$$

There are three forms of condenser of practical utility :—

(1) *The parallel plate condenser.*—This consists of two conducting plates separated by a thin dielectric. Parallel plate condensers of considerable capacity are made by interleaving thin sheets of mica with sheets of tinfoil ; the sheets of tinfoil should be smaller all round than the mica.

All the alternate sheets of tinfoil are joined together by tongues or strips of tinfoil forming two sets with two poles.

Instead of mica a cheaper form is made by using bank-note paper well soaked in paraffin.

A Leyden jar is really a parallel plate condenser. We can imagine a sheet of glass coated on each side with tinfoil as in Fig. 83, and bent round into a jar as in Fig. 84.

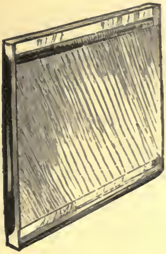


Fig. 83.

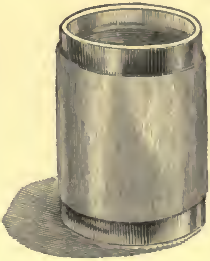


Fig. 84.

(2) **The Spherical Condenser.**—This consists of a metal sphere placed within a hollow metal sphere, and carefully insulated from it. The spherical condenser is shown in Fig. 85, and it is of great historic importance as having been used by Faraday in his researches on specific inductive capacity.

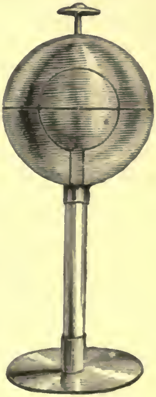


Fig. 85.

(3) **The Cylindrical Condenser.**—This consists of a cylindrical conductor placed inside a hollow cylindrical conductor, from which it is separated by an insulating medium. This is an important form, as cables form cylindrical condensers. The conducting strands of wire are surrounded with insulating material which is protected by an outer coating of metal.

2. **Energy and Capacity of charged Condensers.** — A condenser possesses energy when charged, that is, it is capable of performing work on discharging. If the discharge takes place through a wire, the wire will be heated, and the energy will be thus found to be transformed into heat.

There is every reason to suppose that the energy really resides in the dielectric medium, which we may suppose is in a state of strain; and we may investigate the energy of a cubic centimetre of the dielectric. We will first find the energy of any condenser.

To find the Energy of a Condenser capacity C *charged with* Q *units to a Difference of Potential of* E.

The energy of the condenser may be found by finding the work which must be done to charge it.

Suppose the condenser be charged by carrying over successively a small quantity q units.

Since

$$Q = C.E,$$

the E.M.F. varies as the charge.

∴ if the charge increases uniformly, the work done in carrying over the unit increases uniformly; hence the work done in carrying over each equal quantity q units increases uniformly from zero to E ergs.

Hence we may suppose that in carrying across q units on the average,

$$\frac{E}{2}q \text{ ergs of work are done,}$$

∴ in carrying over Q units,

$$\frac{EQ}{2} \text{ ergs are done,}$$

Hence

$$\text{energy} = \frac{EQ}{2} \text{ ergs} = \frac{CE^2}{2} = \frac{Q^2}{2C}.$$

3. **On a Battery of Condensers.**—Condensers may be connected in two ways, or in a combination of these. They may be joined either in *arc* or in *series*. A combination of condensers constitutes a battery.

(i) *Condensers arranged in arc.*

To arrange condensers in arc, join all of one set of plates together, and all the others.

For example, the condensers A, B, C have all their inner plates connected, and all their outer plates connected; the poles of the combination being *a* and *b*.

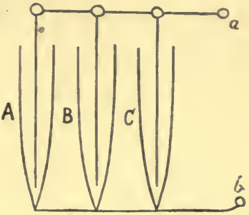


Fig. 86.

To Find the Capacity of the Combination.

Let V_1 be the potential of *a*, and V_2 the potential of *b*.

Then the P.D. of each condenser is $V_1 - V_2$.

Let Q_1, Q_2, Q_3 be the charges of A, B, C respectively, and let C_1, C_2, C_3 be the capacities of A, B, C respectively.

Then

$$Q_1 = C_1(V_1 - V_2)$$
$$Q_2 = C_2(V_1 - V_2)$$
$$Q_3 = C_3(V_1 - V_2)$$
$$\therefore \quad Q_1 + Q_2 + Q_3 = (C_1 + C_2 + C_3)(V_1 - V_2).$$

But if C is the capacity of the combination, its charge is $Q_1 + Q_2 + Q_3$ and P.D. $= V_1 - V_2$;

$$\therefore \quad Q_1 + Q_2 + Q_3 = C(V_1 - V_2);$$
$$\therefore \quad C = C_1 + C_2 + C_3.$$

Or, The capacity of a battery of condensers arranged in arc is the sum of the capacities of each condenser.

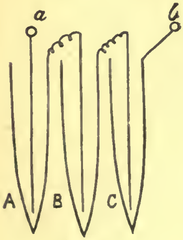


Fig. 87.

(ii) *Condensers arranged in series.*

To arrange condensers in series, join one pole of the first condenser to one of the second, then the remaining pole of the second to one of the third, and so on, leaving one pole of the first *a* and one pole of the last *b*, as poles of the combination.

To find the Capacity of the Combination.

If the condensers are either so far separated that they produce no inductive effect on each other, or if they are of a cylindrical form with inside plates, then if

Q is the charge on the inner plate of A, $-Q$ is the charge on the outer plate of A, and $+Q$ the charge on the inner plate of B (see p. 173, iv.). Hence the charge of each condenser is the same.

Let

E_1 be the potential difference of A, and C_1 the capacity;
E_2 ,, ,, B, and C_2 ,,
E_3 ,, ,, C, and C_3 ,,

Then

$$Q=C_1E_1$$
$$Q=C_2E_2$$
$$Q=C_3E_3$$

Now the P.D. of the combination is the sum of the P.D.'s of each.

∴ The P.D. of the combination is $E_1+E_2+E_3$.

If C is the capacity of the combination, Q is the charge (if *a*, *b* are joined, Q will pass across).

$$\therefore \quad Q=C(E_1+E_2+E_3);$$
$$\therefore \quad \frac{Q}{C}=\frac{Q}{C_1}+\frac{Q}{C_2}+\frac{Q}{C_3};$$
$$\therefore \quad \frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}.$$

Or, **The reciprocal of the capacity of a battery of condensers in series is the sum of the reciprocals of the capacities of the condensers.**

EXAMPLE 1.—Find the capacity of a battery of 6 condensers arranged 2 sets, of 3 in series, in arc. The capacity of each being 2 microfarads.

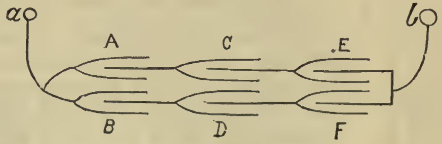


Fig. 88.

The capacity of the three in series is given by

$$\frac{1}{C_1}=\tfrac{1}{2}+\tfrac{1}{2}+\tfrac{1}{2} \quad \therefore \; C_1=\tfrac{2}{3};$$

$\therefore$ The capacity of each set of 3 in series is $\frac{2}{3}$;

$\therefore$ The capacity of the combination of the two rows arranged in arc $=\frac{4}{3}=1\frac{1}{3}$ microfarad.

EXAMPLE 2.—Find the capacity of the condensers A, B,

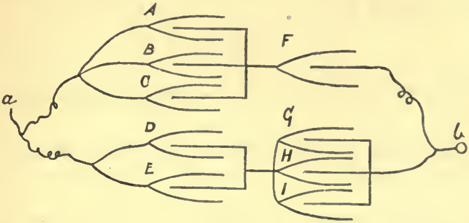


Fig. 89.

C, D, E, F, G, H, I, the capacities of which are $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, 1, 2, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, 1, arranged as in the figure.

Capacity of ABC $=\frac{1}{2}+\frac{1}{3}+\frac{1}{4}=\frac{6+4+3}{12}=\frac{13}{12}$.

$\therefore$ Capacity of ABCF is given by

$$\frac{1}{C}=\frac{1}{\frac{13}{12}}+\frac{1}{\frac{1}{2}}=\frac{12}{13}+2=\frac{38}{13} \quad \therefore C=\frac{13}{38}.$$

Capacity of DE $=1+2=3$.

Capacity of GHI $=\frac{1}{3}+\frac{1}{5}+1=\frac{5+3+15}{15}=\frac{23}{15}$.

$\therefore$ Capacity of DE and GHI in series is given by

$$\frac{1}{C}=\frac{1}{3}+\frac{1}{\frac{23}{15}}=\frac{1}{3}+\frac{15}{23}=\frac{68}{69} \quad \therefore C=\frac{69}{68}.$$

Hence capacity of ABCF and DEGHI in arc

$$=\frac{13}{38}+\frac{69}{68}=1.356.$$

4. **Capacities of Condensers in Absolute Units.**—For the proofs of the following facts the student may consult more advanced text-books :—

(1) The capacities of condensers in absolute electrostatic units are given by the following formulæ :—

(*a*) Parallel plate condenser, the area of the plates S sq. cm. ; distance between them, *i.e.* thickness of the dielectric d cm. ; specific inductive capacity K.

$$C = \frac{K.S}{4\pi d}.$$

(*b*) Spherical condenser ; radii a, b cm. respectively ; specific inductive capacity K.

$$C = \frac{Kab}{b-a}.$$

It follows from this that if b is made very large,

$$C = Ka,$$

i.e. the capacity of a sphere placed in a large room $= Ka$, *i.e.* varies as its radius.

(*c*) Cylindrical condenser, length l cm. ; radii of surfaces a and b cm. ; specific inductive capacity K.

$$C = \frac{Kl}{2 \log_e \frac{b}{a}}.$$

Now it will be shown below that

1 electrostatic unit (E.S.U.) of capacity $=$

$\frac{1}{v^2}$ electromagnetic unit (E.M.U.) of capacity ;

and 1 microfarad $= 10^{-15}$ E.M.U. of capacity ;

$$\therefore \quad 1 \text{ E.S.U. of capacity} = \frac{10^{15}}{v^2} \text{ microfarad} ;$$

$$= \frac{10^{15}}{(3 \times 10^{10})^2} \text{ microfarad} ;$$

$$= \frac{10^{15}}{9 \times 10^{20}} = \frac{1}{900000} \text{ microfarad.}$$

Hence the capacity of a given condenser can be obtained in microfarads by dividing the expression given above by 900,000.

Ex. 1.—Find in microfarads the capacity of a plate condenser 50 cm. sq., and dielectric thickness 1 mm., the S.I.C. being 6.

$$C = \frac{KS}{900000 \times 4\pi d} \text{ microfarads} ;$$

$$= \frac{6 \times 50^2}{900000 \times 4\pi \times .1} \text{ microfarads.}$$

Ex. 2.—Find the area of a plate condenser which shall have a capacity of 1 microfarad, given that the thickness is $\frac{1}{4}$ mm. and S.I.C. $=4$.

$$C=\frac{KS}{4\times 900000\times \pi d};$$

$$\therefore \quad 1=\frac{4\,.\,S}{4\times 900000\times \pi\times \frac{1}{40}};$$

$$\therefore \quad S=\frac{90000\pi}{4}=70686.$$

Hence the plate would be a square of about 265 cm. side.

(2) The tension exerted on a sq. cm. of a charged surface is given in dynes per sq. cm. by

$$p=\frac{2\pi\sigma^2}{K}.$$

Hence the force on a plane area of area S sq. cm. is given by

$$P=\frac{2\pi\sigma^2 S}{K}\text{ dynes};$$

σ being the density at a point of the surface in absolute electrostatic units per sq. cm.

A parallel plate condenser where the distance between the plates is d cm. and the E.M.F. E, will have a charge given by

$$Q=CE,$$

where

$$C=\frac{KS}{4\pi d};$$

$$\therefore \quad Q=\frac{KSE}{4\pi d};$$

but

$$Q=\sigma S,$$

$$\therefore \quad \sigma=\frac{KE}{4\pi d};$$

$$\therefore \quad P=\frac{2\pi}{K}\cdot\frac{K^2E^2S}{16\pi^2d^2};$$

$$=\frac{KE^2S}{8\pi d^2}.$$

5. **Electrostatic Measure of E.M.F.**—The force which a charged plate exerts on a parallel plate, the area

of which is S sq. cm., and distance d cm. is, when the dielectric is air, found to be $\frac{S}{8\pi} \cdot \frac{E^2}{d^2}$ dynes. This gives a method of measuring E in absolute electrostatic units. In order that the conditions stated in case (3) above, viz. that the lines of force between the plates should be parallel, Sir Wm. Thomson has constructed the plates in the following manner:—AB, CD are the two parallel plates. Out of the plate AB a circular disc F is cut, and this is the plate which is capable of motion. When F is in position in the plane AB, the lines of force falling upon it are perpendicular to it. The annulus surrounding it is called a *guard ring*.

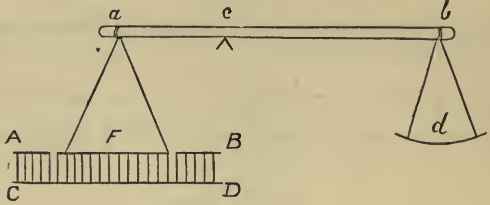


Fig. 90.

If now F is suspended from one end a of a beam capable of turning about a fulcrum at c, weights may be suspended from a scale pan at the end b.

If now F and C be kept at constant E.M.F. by being joined to the poles of a battery, F will be pulled down by the force $\frac{SE^2}{\pi d^2}$ dynes, and by placing weights in the scale pan, the force along aF can be made to balance this force of attraction, when the disc is in position.

An instrument constructed on these principles, with all precautions for insulation, and for measurement of the force, and for adjustment, forms *an Absolute Electrometer.*

The following experiment was made by Sir Wm. Thomson. He joined 1000 Daniell cells in series, and connected one pole to the disc, and the other to the lower plate. He found that with plates 100 sq. cm. in area, and distance apart of 1 mm., that the force of attraction was 5.7 gm. wt.

If the value of g is 981 cm. per sec. per sec. 5.7 gm.wt. = 5.7 × 981 dynes.

Now

$$P=\frac{E^2S}{8\pi d^2}$$

$$\therefore\ 5.7\times 981=\frac{E^2\times 100}{8\times 3.1416\times(.1)^2}$$

$$\therefore\ E^2=\frac{5.7\times 981\times .01\times 8\times 3.1416}{100}$$

$$\therefore\ E=3.75.$$

Therefore the E.M.F. of 1000 Daniell cells in series is 3.75 Absolute Electrostatic units.

Hence E.M.F. of 1 Daniell cell = .00375 absolute electrostatic unit.

This experiment is of great importance to us, as it is a direct experiment which enables us to find the number of electrostatic units of electricity in one electromagnetic unit.

The E.M.F. of a Daniell cell has been determined by Sir Wm. Thomson to be 1.07×10^8 absolute electromagnetic units.

Hence, comparing the following statements, we are able to compare the two units of quantity :—

i. When one electrostatic unit passes through a Daniell cell .00375 erg of work is done.

ii. When one electromagnetic unit passes through a Daniell cell 1.07×10^8 ergs of work is done.

Hence the electromagnetic unit is very much larger than the electrostatic unit, for we see that it must contain it $\frac{1.07\times 10^8}{.00375}$ times, *i.e.* 2.86×10^{10} times, approximately 3×10^{10} *i.e.* 30,000 million times.

The number 3×10^{10} is generally denoted by v. It is approximately in cm. per. sec. the velocity of light.

The electromagnetic unit of quantity of electricity $= v \times$ electrostatic unit of quantity.

Now the

$$\begin{aligned}\text{E.M.U. of E.M.F.} &= \text{erg per E.M.U. of Quantity.}\\ &= \text{erg per } v \text{ E.S.U of Quantity.}\\ &= \frac{1}{v}\text{ erg per E.S.U. of Quantity.}\end{aligned}$$

But the

erg per E.S.U. of Quantity is the E.S.U. of E.M.F.

$$\therefore\quad \text{E.M.U. of E.M.F.} = \frac{1}{v}.\ \ \text{E.S.U. of E.M.F.}$$

Again

$$\text{E.M.U. of Quantity} = v \times \text{E.S.U. of Quantity},$$

and

$$\text{E.M.U. of E.M.F.} = \frac{1}{v} \times \text{E.S.U. of E.M.F.}$$

$$\therefore \quad \text{E.M.U. of Capacity} = \frac{v}{\frac{1}{v}} \times \text{E.S.U of Capacity}$$

$$= v^2. \quad \text{E.S.U. of Capacity.}$$

$$\therefore \quad \text{E.S.U. of Capacity} = \frac{1}{v^2}. \quad \text{E.M.U. of Capacity.}$$

We have used this result on page 192 to find the capacity of a condenser in microfarads.

EXERCISES

1. Define capacity, potential, charge.

A metal disc is insulated and charged. A second disc is connected to earth and gradually brought near to the first, parallel to it. Explain, as regards the first disc, in what manner the electrical quantities named above change with the approach of the second disc. What further difference will be produced if a sheet of glass is placed between the discs? How would you support your statements by experiment?

2. Describe an electrical condenser and explain its action. By what experiments can it be proved that the charge of a condenser is directly proportional to the P.D. of its plates?

3. State the chief forms of condensers, and show how to construct a paraffin paper condenser.

4. What is a Farad? a microfarad? Show that the microfarad is 10^{-15} of the E.M.U. unit of capacity.

5. Condensers of respectively 1, $\frac{1}{2}$, $\frac{1}{3}$ microfarads are arranged (i) in series, (ii) in arc; find the capacities of the two batteries.

If the first two in arc are arranged in series with the last, what is the capacity of the battery?

6. Find the capacity of a parallel plate condenser, area of each plate 100 sq. cm., dielectric thickness .5 mm., S.I.C. of dielectric 6.

7. Find the capacity of a Leyden jar 20 cm. high, 10 cm. radius, thickness of glass .5 mm. (i) not including (ii) including the base, and compare the capacity with a microfarad.

8. A sphere of 10 cm. radius is placed within a hollow concentric sphere of 20 cm. radius—dielectric being air—find its capacity.

Find the capacity if the sphere is placed in a very large room.

Compare each capacity with the microfarad.

9. In Questions 6 and 7 find the force between the two plates (i) when charged to E.M.F. of 1000 volts, (ii) when charged with 1000 coulombs.

10. Describe the principle of the absolute electrometer, and indicate the method by which the two units of quantity can be compared.

SECTION VII

To Compare the Capacities of Condensers

1. From the relation $Q = C.E$ it is seen that if we have either (i) means of charging two condensers with known charges and then measuring their P.D.'s; or (ii) means of charging two condensers to known P.D.'s and measuring the charges, then we can compare their capacities.

The first is the method generally adopted in electrostatics (using electrometers) and the second that adopted in electromagnetic methods (using galvanometers).

i. (1) The capacities of condensers of the nature of Leyden jars can be compared by charging them with equal charges, by discharging within each the charge from a ball charged successively with equal charges, and then comparing the P.D.'s by the electrometer. If the charges are equal the capacities are inversely as the P.D.'s. The difficulty here is to charge the condensers with *equal* quantities—the following method is the ingenious method of surmounting this difficulty.

(2) *Faraday's Method.*—Place the two condensers on an insulating stand, and connect their outer coatings to the earth.

Now charge one of them, suppose its charge is Q_1. Let C_1 be its capacity and C_2 the capacity of the other.

By means of the electrometer measure the potential difference of the condenser. Suppose the deflection is δ, then

$$Q_1 = C_1 E_1 \quad (\alpha)$$

Now join the two inner coatings, thus forming one condenser. If C_2 is the capacity of the second condenser,

$$C = C_1 + C_2$$
$$\therefore \quad Q_1 = (C_1 + C_2)E_2 \quad (\beta)$$

Measure the P.D.'s of the combination by the electrometer. Let the deflection be δ_2, then $E_2 = k\delta_2$.

From α and β

$$C_1E_1 = (C_1 + C_2)E_2 \qquad \therefore \quad \frac{C_2}{C_1} = \frac{E_1 - E_2}{E_2} = \frac{\delta_1 - \delta_2}{\delta_2}.$$
$$\therefore \quad C_2E_2 = C_1(E_1 - E_2)$$

ii. The second method is suitable for condensers of large capacities, comparable with the microfarad, and is a galvanometer or electromagnetic method.

All that is necessary is to take a cell of constant E.M.F.—a standard Daniell cell, and charge each condenser by means of it. By using a suitable key (called the Morse key) the condenser can be charged and discharged through a galvanometer, the throw of which will be proportional to the quantity of electricity required to charge the condenser to the E.M.F. of the Daniell cell.

If the throws for the two cells are respectively θ, and θ^1, then the charges are proportional to θ and θ^1. But the capacities are proportional to the charges, if the E.M.F.'s are the same. Hence the capacities are proportional to the throws.

2. **Influence of the dielectric. Specific inductive Capacity.**—In Section V. i. we have seen that the introduction of a sheet of glass, vulcanite, shellac, etc., diminishes the P.D. for the same charge—*i.e.* it increases the capacity of a condenser. Faraday was the first to discover the influence of the dielectric, and he used the method above for comparing the capacities of two condensers which were similar and equal in all respects, except in one case air, and in the other some other insulator, was the dielectric. He found that the ratio of the capacity of a shellac condenser, to a similar and equal air condenser, was a constant depending only on the nature of the dielectric. This constant is called the specific inductive capacity of the dielectric.

For our present purpose the following definition may be taken :—

DEFINITION.—**The specific inductive capacity of a dielectric is the ratio of the capacity of a condenser with the given dielectric as insulating medium, to the capacity of a similar and equal condenser in which air is the insulating medium.**

Methods of finding the specific inductive capacity

i. *Faraday's Method.*—Faraday constructed two identical spherical condensers as in Fig. 85, p. 187, and in one air, in the other shellac, was the dielectric. He then compared their capacities by the method above. If K is the specific inductive capacity of the insulating medium, C_2 the capacity of the condenser with this medium as dielectric, and C_1 the capacity of the air condenser. Then

$$K=\frac{C_2}{C_1}=\frac{\delta_1-\delta_2}{\delta_2}.$$

ii. *Cavendish's Method.*—Cavendish constructed a condenser by placing one tin cylindrical vessel within another. He then found the thickness of a parallel plate air condenser of given plates, which had the same capacity as this when air was the dielectric, and second when the dielectric was some other insulator.

EXERCISES

1. Give Faraday's method of comparing the capacities of condensers, and show how he compared the S.I.C. of dielectrics by this method. Could Faraday's method be used to compare condensers commensurable with the microfarad? Give reasons.

2. What is meant by specific inductive capacity? What is the principle of the method adopted by Cavendish to compare the S.I.C.'s of two dielectrics?

EXERCISES ON CHAPTER III

1. Describe briefly the general phenomena of electrostatic induction.

2. A plate of brass and a plate of ebonite are placed on a table and struck with a catskin. What will happen in each case? What would happen if the plate of brass were placed on the top of the ebonite and then struck with the catskin?

3. Three insulated metal balls A B and C are placed in a line, A and B in contact, C a little way off. C is positively electrified, and then A and B are separated. What are now the electrical states of A and B?

4. How is it that in damp weather an ordinary plate electrical machine will not work well?

5. A piece of brass tube when struck with a catskin and made to touch an electroscope shows no sign of electrification, how would you prove experimentally that it was really electrified when so struck?

6. Draw a diagram illustrating the action of an electrophorus, and state from what source the electric energy is derived.

7. What is an electrophorus, and how is it used? Give a reason for each part of the process.

8. Describe some simple form of electroscope. What is the use of surrounding the essential portions with wire gauze?

9. State exactly what must be done in order to get a series of sparks from an electrophorus.

10. A small pith ball rests on a brass plate provided with a glass handle. The two are placed on a cake of resin previously rubbed with catskin. The plate is touched with the finger and then removed, why should the pith ball jump off the plate?

11. Two small balls are charged respectively with $+48$ and -8 units of electricity, with what force will they attract one another when placed at a distance of 4 cm.?

12. Describe any means of observing the electrical condition of a small body without altering the body's condition.

13. Discuss the evidence in favour of the view that the force in play between two electrical charges (each supposed concentrated at a point) varies inversely as the square of the distance between them.

14. Describe carefully Faraday's ice-pail experiment. What deductions do you draw from the experimental facts observed?

15. What is meant by a "unit of electricity"? Explain the statement "the charge resides on the surface of conductors," and describe the experimental facts summarised in it.

16. How does the attraction between two small electrified balls vary with the distance between them? How may this law be verified? What is the unit of electricity adopted in electrostatic measurements?

17. Water is allowed to fall, drop by drop, from a tube connected with an insulated vessel. The knob of a charged Leyden jar is placed near the end of the tube from which the water drops, but so that the water does not touch it. What will be the effect on an electroscope connected with the water vessel? On what does the ultimate electrical condition of the water vessel and electroscope depend?

18. Under what conditions is it possible to transfer the whole of the charge on a conductor to another insulated conductor?

19. An electroscope is surrounded by a cylinder of wire gauze which is put to earth. If an electrified body is brought near to it, how will the leaves behave? Give reasons for your answer.

20. Two pith balls, one suspended by a damp cotton thread the other by a dry silk thread, are each of them touched by the knob of a charged Leyden jar, which is held in the hand by its outer coating, Will there be any difference between the behaviour of the two balls? If so what difference? and why?

21. Two metal balls are placed at a small fixed distance from each other, one being connected with the earth and the other with the prime conductor of an electrical machine kept turning at a uniform rate. How, and why, will the force between the balls be altered if a thick plate of unelectrified non-conductor be placed between them?

22. Two equal insulated metal balls are placed near each other. One of them receives a charge of positive electricity. Show by a diagram the distribution of the lines of electric force in the neighbourhood of the balls.

23. A gold-leaf electroscope is put inside a tin can, which is hung up by dry silk cords. On holding a strongly electrified glass rod below the can, no divergence of the gold leaves takes place ; but on touching the cap of the electroscope with the finger (without touching the can) the leaves diverge. Explain these results.

24. A deep metal pot is placed on an insulating stand and electrified,

a metal ball hung by a silk thread is put in contact with a gold-leaf electroscope after being made to touch—

(*a*) First the *inside* and then the *outside* of the pot
(*b*) First the *outside* and then the *inside* of the pot.

State and explain the effect on the electroscope in each case.

25. What is the "specific inductive capacity" of a substance? How can the Sp. Ind. Cap. of different substances be compared?

26. What is meant by saying that two conductors are at the same, or at a different potential? Show how a conductor might have a + charge and yet be at a − potential.

27. Explain the following terms: difference of potential, charge, density, tension; and describe experiments to illustrate the distinction between (1) potential and charge, (2) potential and density, (3) potential and tension.

28. Why should an electrical machine have its conductors all rounded, except those nearest to the plate or cylinder which are pointed?

29. What is meant by electrostatic potential at a point in an electric field? How would you proceed to determine the potential of an insulated charged conductor?

30. Of two insulated metallic plates A and B, which are placed near and parallel to each other, A is connected with the cap of an electroscope, and B receives a charge of electricity. Explain the behaviour of the gold leaves of the electroscope when a slab of unelectrified sulphur is introduced between the two plates.

31. In some forms of electrometer there is a movable disc surrounded by a wide flat ring in the same plane as the disc. Explain the use of this ring.

32. Two insulated brass plates, a good way apart, are connected by separate wires with a gold-leaf electroscope, and the electroscope and brass plates are electrified so that there is a small divergence of the gold leaves. How and why is the divergence of the leaves altered when the plates are brought near together and facing each other?

33. Two Leyden jars charged in the ordinary way are held, one in each hand, by the outer coatings. What takes place when the knob of the one is made to touch the outer coating of the other, and what is the subsequent condition of each jar?

34. One pole of a battery of many cells is earth connected, and a long insulated wire projects from the other end. Two insulated metal balls, of 1 inch and 5 inch diameter respectively, are put one after the other in contact with the end of the insulated projecting wire. What are the comparative quantities and densities of the electricities on the two balls?

35. Three equal similar condensers are connected—

(1) In series.
(2) In arc or abreast.

They are in each case charged as fully as possible by the same machine. What proportion does the heat produced by discharging the jars in the first case bear to that produced in the second case?

36. A Leyden jar has a coated surface of 100 sq. cm., the Sp. Ind. Cap. of the dielectric is 1.9 and its thickness 0.2 cm. What would be

the diameter of that insulated metal sphere which would have the same capacity as the jar?

37. The internal radius of a Leyden jar is 4.7 cm., the inside coating is 15 cm. high, the glass of the jar is 0.2 cm. thick, and has specific inductive capacity 6. If this jar be charged to potential 300, show how much heat will be produced by its discharge.

[Mechanical equivalent of heat = 42 millions.]

38. Two insulated and widely separated metallic spheres receive charges of positive electricity which raise their potentials to 4 and 5 respectively. The densities of the charges being in the ratio of 4 : 9, compare the radii of the two spheres.

39. Describe and explain a method of comparing experimentally the capacities of two condensers whereby the ratio of the capacities is given as the ratio of two resistances.

40. A Franklin's pane is formed by pasting two sheets of tinfoil, each of which is 100 sq. cm. in area, on opposite sides of a sheet of glass $\frac{1}{2}$ mm. thick. How many sheets of tinfoil of the same size would have to be pasted on opposite sides of a sheet of ebonite 5 mm. thick to make a condenser of the same capacity. The Sp. Ind. Cap. of glass and ebonite being 3 and 2 respectively.

41. Two equal soap-bubbles, equally and similarly electrified, coalesce into a single larger bubble. If the potential of each bubble while at a distance from the other and from all other conductors was P, what is the potential of the bubble formed by their union? (*N.B.*—Volume of a sphere is proportional to cube of the radius $v = \frac{4}{3}\pi r^3$.)

42. Describe and explain an accurate method for measuring the specific inductive capacity of an insulating material.

43. The outside, earth connected, coating of a charged Leyden jar, A, is connected by a wire with the outside coating of an uncharged Leyden jar, B, and the knob of A is then made to touch the knob of B, so that the original charge of A is now shared between the two jars. What would you require to know about the jars in order to be able to calculate the proportion in which the charge is shared? Show how to find the proportion when all needful particulars are given.

44. Describe a method of determining, in absolute measure, the capacity of a given condenser.

45. An insulated metal ball of 9 inches radius, connected by a long fine wire with the prime conductor of an electrical machine at work, takes a charge of 1000 units, when it is at a distance from all other conductors. Show what charge the ball will acquire from the same machine if it is surrounded by a concentric metal cover, connected with the earth, and of 10 inches internal radius.

CHAPTER IV

ELECTROMAGNETISM

SECTION I

Magnetic Field surrounding a Current Circuit

WHEN a current of electricity is passing through a wire, it is found that a magnetic field exists in the neighbourhood of the wire—in fact we shall see that a current circuit carries with it a definite magnetic field, and may be regarded as having tubes of magnetic induction attached to it. We should therefore expect that this current circuit would be acted upon by a magnetic field—whether this magnetic field be due to a permanent magnet or to another current circuit. The part of our subject which investigates the mutual action of current circuits, and a magnetic field is called electromagnetism. If the student is familiar with the magnetic fields belonging to the more usual circuits, and with the action of magnetic fields on each other as explained in Chapter I., he will have no difficulty in understanding the following sections. Before considering these mutual actions the student must investigate the fields of force accompanying (1) the circular circuit, (2) the solenoidal or spiral circuit, (3) the straight circuit, *i.e.* the field of force in the immediate neighbourhood of a small portion of a closed circuit. These fields have been explained in Chapter II. § 2, and we will now give a general law by means of which these fields can be predicted, a law by which the mathematician can determine the intensity at any point of the field due to each of the above circuits.

Ampère's Law.—Ampère has shown that a current in a circuit acts and is acted upon as if it were a magnetic shell, the boundary of which coincides with the circuit, one surface of which is magnetised with north magnetism, and the other with south magnetism ; and that the north and south faces of the shells are related to the direction of the current according to the following rule : Imagine a right-handed screw twisted in the direction of the current so that it pierces the coil at right angles to its plane, then the direction of translation of the screw is from the south towards the north of the equivalent magnetic shell. This is Ampère's Rule, and the law upon which it is based is called Ampère's Law (this is given on p. 205).

An example of this rule may be given : Hang a watch up with its face towards you, and imagine an insulated wire placed round its edge, and suppose a current to pass round it in the direction of rotation of the hands of the watch. This current circuit will behave as a magnetic shell or disc, one face being north, the other south. To find the north face of the shell imagine a right-handed screw to be screwed through the face of the watch, the direction of twist being the direction of the current and therefore of the hands of the watch, then the direction of translation will be from the face towards the back, and as by the Rule it is from the south to the north, the equivalent shell must be magnetised with south magnetism on the face, and north magnetism on the back, *i.e.* the axes of magnetisation must be perpendicular to the plane of the coil, and from back to face. If now a small suspended magnet be placed in front of the watch, the north pole will be attracted towards the face. Hence the lines of force due to the current circuit run through from the front to the back. Again consider the solenoidal spiral. Each circuit of this spiral can be replaced by a thin shell. Hence the whole solenoid — as far as external points are concerned—can be replaced by a number of thin magnetised discs placed with their north and south faces touching each other. A number of such discs placed face to face will form a bar magnet with one end north, the other south, and the lines of force will run in at the south end and come out at the north end. Hence if we twist in the direction of the current, the direction of translation will be from south to north of the equivalent magnet.

Note.—The intensity at any point of the field can be determined experimentally either by the method of deflection $F = H \tan \delta$ or by oscillation $F \times \frac{n'^2}{n^2}H$. The student may verify the following formulae which have been obtained by a mathematical application of Ampère's Law.

(*a*) The intensity at the centre of a plane circular coil of n turns and r cm. radius is given by

$$F = \frac{2\pi n}{r} i.$$

(*b*) The intensity at any point within a straight spiral, where the field is uniform, is given by

$$F = 4\pi n_1 i$$

where n_1 is the number of turns of the spiral per cm. of length of the spiral. If N = total number of turns of the spiral and l cm. its length, then $n_1 = \frac{N}{l}$.

(*c*) The intensity at a point near a straight circuit and distant d from it is given by

$$F = \frac{2\pi i}{d}.$$

It will be well to state Ampère's Law more exactly, and we will give it in the form adopted by Chrystal in the *Encyclopædia Britannica.*

"It results alike from the fundamental experiments of Ampère and the elaborate researches of Weber, that an electric current circulating in a *small* plane closed circuit, acts and is acted upon magnetically exactly like a small magnet placed perpendicular to its plane at some point within it, provided the moment of the magnet be equal to the strength of the current multiplied by the area of the circuit, and its N. pole be so placed that the direction of the axis of the magnet (from S. pole to N. pole), and the direction in which the current circulates, are those of the translation and rotation of a right-handed screw which is being screwed in the direction of the axis."

If, therefore, the area of the *small* circuit is a sq. cm., and the current is i units, then the magnetic moment of the equivalent magnet is *ia* units. Consider any circuit ABC. Imagine it filled with a surface of any form and a network of lines drawn on the surface as in the figure, dividing it up into portions such as *a*, *b*, *c*, *d*, so that they may be regarded as *plane*. It is obvious that any current of strength *i* circulating in A B C may be replaced by a series of closed circuits circulating in the meshes (such as *a b c d*) of the network on the surface; for in each line such as *bc* we have two equal and opposite currents circulating whose action must be nil. Now we may replace each of these small circuits by a magnetic shell of strength *ia*. The magnetic action of this shell at every point external to it will be the same as the magnetic action of the circuit. The north side of the shell is derived from the direction of the current by the right-handed screw relation.

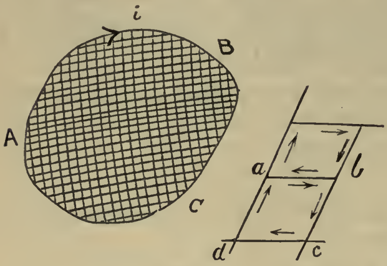


Fig. 91.

Examples of this will now be given :—

(1) A plane circular circuit of one turn of radius *r* having a current *i* circulating in it. This can be replaced by a plane circular network of say equal meshes (of area *a*), having a current *i* circulating in each in the same direction. Each mesh can be replaced by a magnet of magnetic moment *ia*, having its axis perpendicular to the plane of the mesh, *i.e.* perpendicular to the plane of the coil. Hence the circular circuit can be replaced by a magnetic circular disc having a magnetic moment equal to the sum of the magnetic moments of the small magnets, *i.e.* equal to *i*A where A is the area of the whole circuit. The axis of each magnet being perpendicular to the plane of the coil, the axis of the equivalent disc will be perpendicular to the coil, and the direction of the axis will be given by the right-handed screw law.

(2) The spiral in which a current is circulating can be replaced by a series of magnetic shells forming a solenoidal magnet.

EXERCISES

1. Sketch the lines of force (i) due to a circular current circuit, (ii) due to a spiral current circuit, (iii) in the immediate neighbourhood of the straight part of a circuit. How do these fields differ from those due to the equivalent magnets?

2. State Ampère's Law as regards the nature of the equivalent shell, and show how to deduce from it the fact that any current circuit can be replaced by a certain magnetic shell.

3. Use Ampère's Law to show that a spiral current circuit acts as a solenoidal magnet, and find which is the north end of the magnet.

SECTION II

On the Action of a Magnet Field on a Current Circuit

1. In Section I. we have seen that a circuit in which a current is passing acts as if it were a magnetic shell constructed according to Ampère's Rule. We have now to consider how such a current circuit will behave when placed in a magnetic field, and the student will be able to verify the second part of Ampère's Law, viz. a current circuit is acted upon precisely the same as if it were a magnetic shell constructed according to Ampère's Rule.

There are two methods of determining the direction in which a magnet will set in a field.

(i) It will set with its equivalent axis in the direction of the field, hence we must find the direction of the axis, and the relative position of its north and south poles. This in the simple case of the disc and solenoidal magnet is easily done. In the case of a plane disc, the axis is perpendicular to the plane, and in the case of a solenoid the axis is in the direction of the axis of the spiral of the solenoid.

(ii) In the chapter on magnetism it is shown that a magnet will set in a field so that it disturbs the original field as little as possible, or so that as many lines of force as possible pass through the magnet in the direction of its axis.

We have, therefore, two methods of determining the action of a magnetic field upon a coil:

(1) *By Ampère's Law.*—Replace the current circuit by its equivalent magnet according to Ampère's Law, and then

consider how it will be acted upon when placed in the given field.

(2) *Maxwell's Law.*—Since a current circuit according to Ampère's Law can be replaced by a magnet, and since a magnet will set so that as many lines of force as possible pass through it, we can apply a rule due to Maxwell and called Maxwell's Law.

A current circuit placed in a magnetic field will experience forces tending to set it so that it will embrace as great a number of lines of force as possible, and if free to move it will set in the position in which it embraces the greatest number of lines of force.

The following experiments will enable the student to test the truth of Ampère's Law and Maxwell's Law.

Expt. 1.—Make a circular coil about 10 cm. diameter and of 10 turns, and bind the turns together with cotton. Bend one end of the coil in the direction of a diameter of the coil, and the other end bend three times at right angles, as shown in the figure. Turn a circular trough in a piece of wood, and divide the trough into two semicircles by means of small pieces of cork at *c* and *c'*. Fill the trough with mercury, so that the surface of the mercury stands above the corks. Now suspend the coil by a few silk fibres so that one end of the coil dips into mercury placed in a circular hole at the centre of the trough, and the other end dips into the mercury in the circular trough. Now join the central hole to one pole of a Bunsen's cell, and the trough to the other pole, so that a current circulates through the coil, and the coil itself is free to turn about a vertical axis. With a little care in the adjust-

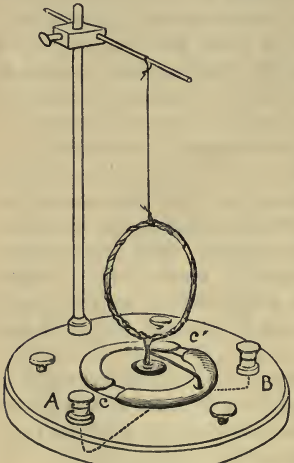


Fig. 92.

ment it will be found that the coil turns so that its plane is perpendicular to the meridian, and in such a position that, if we screw perpendicularly to the coil in a right-handed direction from S to N, the direction of rotation of the screw will be the direction of the current. Maxwell's rule is also obeyed because since the coil turns so that its plane is at right angles to the magnetic field it will embrace as many lines of force as possible.

If a bar magnet be presented to the coil it will be found that Ampère's and Maxwell's Laws are obeyed. The coil will always set at right angles to the lines of force (see Fig. 103, and *cp.* iv. p. 220), and the direction of the field will be the direction of translation of the screw if it is twisted in the direction of the current.

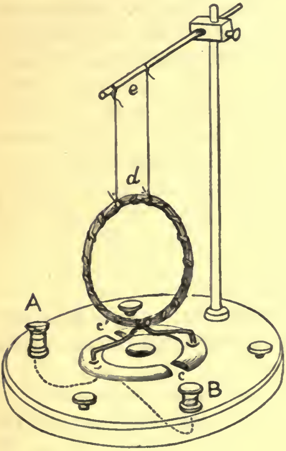


Fig. 93.

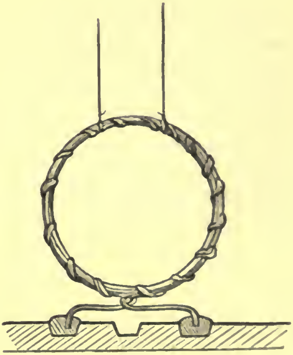


Fig. 94.

It will also be found that the coil is repelled or attracted according to the same laws.

For this experiment it is better to suspend the coil by two parallel fibres, as in Fig. 94.

Place a north pole in front of the coil which is supposed to be in the plane of the paper, so that lines of force will run through the coil from the front to the back of the paper. If now the direction of the current in the coil is right-handed as we screw through the paper in the direction of the lines of force from the front to the back, according to Ampère's Law (as we screw from S to N of the equivalent magnet) the equivalent magnet will have its S face towards the N pole of the presented magnet. Hence the suspended coil will be attracted, and thus will embrace more lines of force, according to Maxwell's Law.

Numerous experiments of the same kind can be performed, all verifying Ampère's Law, and the consequent law of Maxwell. The same form of apparatus can be used to study the behaviour of a solenoidal spiral when capable of turning about a vertical axis. A spiral of wire can be wound and its two ends turned to dip into the mercury. It will be found that the spiral behaves as a bar magnet and sets with its axis along a line of force, and consequently with the plane of each coil perpendicular to the lines of force, and so that if we screw along a line of force, *i.e.* in the direction in which a north pole will move, the direction of translation will be along the axis of the spiral, and the direction of twist will be the direction in which the current is circulating through the spiral.

2. On the action of a magnetic field on a straight circuit, *i.e.* on a straight portion of a circuit.

Let the straight portion of the circuit be in the plane of the paper, and suppose the current to run upwards as shown by the arrow in the figure.

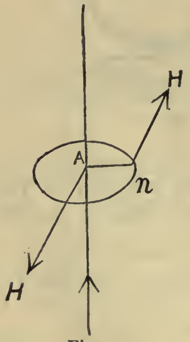


Fig. 95.

The lines of force due to the circuit are circular and perpendicular to the wire, *i.e.* in planes perpendicular to the plane of the paper. Hence a north magnetic pole n placed in the plane of the paper to the right-hand side of the wire, as in the figure, will be acted upon by a force perpendicular to the plane of the paper and in a direction through the paper,—according to the rule given in Chapter II. (*i.e.* if we screw along the straight circuit in the direction of the current, the north pole will be acted upon by a force tending to drive it in the direction of the rotation of the right-handed screw).

If now the north pole *n* is fixed, and we assume the third law of motion to be true, the straight circuit will be acted upon by a force parallel to, but in the opposite direction to the force acting on the north pole *n*. Hence the straight circuit will be acted upon by a force perpendicular to the paper and away from the paper towards this side.

Now the action on the straight circuit at A will depend upon the magnetic field at A. This is due to the north pole placed at *n*. Hence the intensity of the magnetic field at A is perpendicular to the circuit, and in the plane of the paper and from the right to the left.

Consider now a straight circuit A*i* placed in a field of intensity H. Let the circuit be in the plane of the paper, and the intensity H perpendicular to the paper and outwards from the paper. Draw AH perpendicular to the paper and produce it through the paper towards N on the other side. The intensity H at A can be supposed due to a *north* pole, placed somewhere along AN. Now such a pole would be set by the current circuit in a direction parallel to the plane of the paper and from right to left. Hence conversely the circuit at A will be acted upon by a force in the plane of the paper and from left towards the right, *i.e.* in the direction of the arrow marked at P, where AP is *in* the plane of the paper.

We see, therefore, that the three directions of *i*, H, and P are related as in the figure.

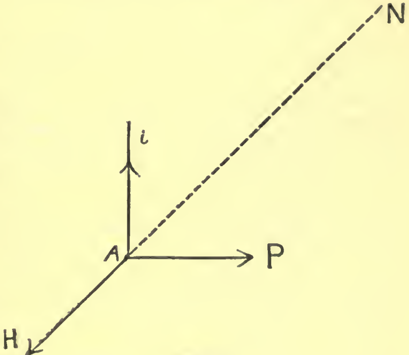


Fig. 96.

The workman's rule can be tested. If the right hand be placed with the first finger pointing in the direction of the current, the second finger bent inwards and pointing in the direction of the field at A, then the thumb bent outwards at right angles to these will point in the direction of the force acting upon A.

(i) The celebrated experiment of Faraday is a verification of this theory.

A thin disc of copper is placed so as to turn smoothly but in metallic contact between the brass fork supports as in the figure. The edge, which is amalgamated with mercury, turns in a groove filled with mercury. A current can be sent from a battery B through the disc from *b* to *a*. A north pole is placed in front of the disc at N. According to our experiment the current flowing upwards from *b* to *a* will exert a force on N from left to right. Hence N being fixed and *ba* free to move, *ba* is turned from right to left. Hence the disc will continue to turn in a right-handed screw direction looked at from the N pole.

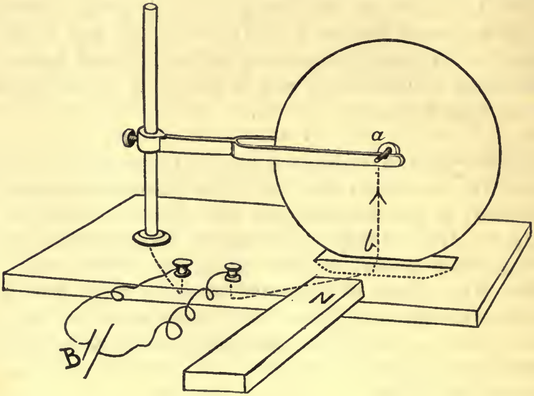


Fig. 97.

(ii) Suppose an oblong circuit *abcd* be placed with its plane in the magnetic meridian, with the ends *bc*, *ad* along lines of force and a current pass from *a* to *b* and from *c* to *d*.

The straight circuit *cd* is acted upon by a force perpendicular to the plane of the paper (which contains the direction of the lines of force and of the current) and towards the other side of the paper. Similarly *ab* is acted upon by a force perpendicular to the plane of the paper and away from the paper. Hence the rectangle is acted upon by a couple tending to turn it about an axis perpendicular to the lines of force. That is, it is acted upon by a couple tending to set it with its plane perpendicular to the lines of force. This is the result obtained by the application of Ampère's Law.

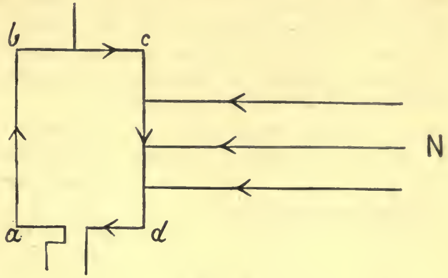


Fig. 98.

If such a rectangle be suspended as in Fig. 92, p. 208, and a north pole of a magnet be brought up to *cd*, the circuit *cd* will be found to begin to turn at right angles to the perpendicular drawn from the north pole to *cd*.

Note 1.—The part of the circuit *bc* which lies along a line of force is not acted upon by any force due to the field.

Note 2.—When the coil is at right angles to the field so that each element of the coil is perpendicular to the lines of force, then each element of the circuit will be acted upon by a force in the plane of the coil and perpendicular to the element, outward if the coil is in its position of stable equilibrium, and inwards if the coil is in its unstable position of equilibrium.

In its stable position the earth's lines (or tubes) of force squeeze through the coil and tend to burst it outwards; in its unstable position the earth's lines are pressed to the outside and tend to surround the coil and thus to compress it inwards. See p. 220 for a further development of this idea.

3. **The Principle of the Electric Motor.**—We have seen that the coil in Fig. 92 turns round until its plane is at right angles to the magnetic field, and oscillates through its position of equilibrium. If now, when it is beyond its position

of equilibrium, the current be reversed through the coil it will continue to turn round in the same direction. We see, therefore, if the current can be reversed each time the plane of the coil is in the field, it will continue to turn, and by the magnetic field exerting a couple upon it the rotatory coil will be able to do work. This is a simple form of electric motor, and if the current is supplied by what is called an alternating dynamo, the motor is an alternating motor.

If, however, the current is a continuous and direct current, it will be necessary to change its direction or commutate it through the coil. This can be done in a simple way by turning the other end of the coil round to dip into the other segment of the circular trough as is done in Figs. 92, 99. Now turn the trough so that the cork gaps in the mercury are in the field. Then each time the coil swings round over the corks the current will be reversed through the coil, so that instead of oscillating about a position of rest, it will continue to rotate. The rotating coil is an elementary "**armature,**" and the mercury ring with the two cork gaps is an elementary "**commutator.**" The field of force may be due to the earth, to magnets, or to electromagnets,—in any case they are called the "**field-magnets.**"

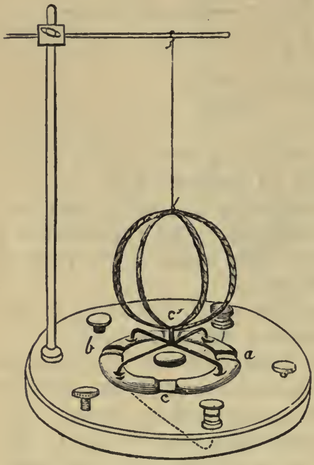


Fig. 99.

In the elementary motor just considered the commutator is fixed, and the sliding contact is made between the armature and the commutator. The usual plan is for the commutator to be fixed and rotate with the armature, the sliding contact being made between the terminals joining the current supplier —these sliding contacts are called **brushes.**

EXERCISES

1. Explain Ampère's method of determining the behaviour of a current circuit free to move in a magnetic field, and apply it to a circular coil suspended about a vertical axis.

2. Apply Ampère's method to determine the behaviour of a cell which has its poles joined by a spiral of wire, and is placed on a cork so as to float in water, keeping the axis of the spiral horizontal.

3. State Maxwell's Law and show how it follows from Ampère's reasoning.

4. Describe a careful set of experiments to verify Maxwell's Law. Give analogies from the floating of bodies in a stream of water.

5. Given the nature of the field of magnetic force in the neighbourhood of a small part of a circuit (*i.e.* a circular or nearly circular field), trace the steps by which the action of the field on the circuit is discovered.

6. Describe Faraday's disc and explain its motion.

7. A coil of wire with a current circulating in it is placed in a parallel magnetic field. State what the mechanical action of the field is, and deduce this action: (i) from Ampère's Law; (ii) directly from Maxwell's Law.

8. Describe a form of an elementary electric motor.

9. State carefully what are the essential elements of an electric motor.

SECTION III

On the Mutual Action of Two Circuits

1. If two plane circuits be placed parallel to each other, and a current be sent through them in the same direction, they can be replaced by two magnetic shells which will have their north and south poles opposed to each other. Hence the two circuits in which the currents pass will attract each other. Conversely, if the currents flow in opposite directions, the coils will repel each other.

For example, if *ab*, *cd* are the two coils, and a current be sent through each of them in the same direction, say in a right-handed screw direction, screwing from below upwards, then in each coil the equivalent magnet will be south on the lower face and north on the upper face. Hence a north and south face are opposed, and the coils will attract each other.

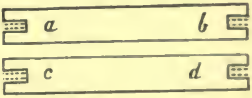


Fig. 100.

The force of attraction will depend upon the square of the current, if it is the same in each. This force of attraction can be measured by having the upper coil suspended from the arm of a balance. This is the principle of the **Kelvin Ampère Balance,** which is adopted as the standard instrument for measuring current. Its value consists in the fact that the only measurement to take is the measurement of weight, no intensity of magnetic field having to be determined.

If the current be sent in opposite directions through the coils, the coils repel each other.

An illustration of this mutual attraction may be given. Make a spiral of copper wire and suspend it vertically from one end so that the other end just dips into a basin of mercury. Now send a current through the coil, from the top through the mercury. The adjacent convolutions will attract each other, and the coil will be shortened and the lower end lifted out of the mercury and contact broken. The spring will then fall and contact will be made again. Hence the wire will vibrate up and down as long as the current is being supplied.

2. **The Action of Parallel Straight Circuits.**—(*a*) Let AB, CD be two parallel straight circuits, in which the currents pass in the same directions. The current circuit AB will give rise to a field of force, the intensity of which at any point P of the second circuit will be perpendicular to the plane of the paper and downwards through the paper. By applying the method of § 2, p. 211, we see that this straight circuit placed in this field will be acted upon by a force in the plane of the paper perpendicular to the circuit CD, and towards the circuit AB. Hence the two circuits will appear to attract each other.

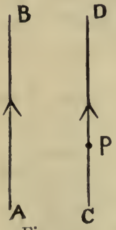


Fig. 101.

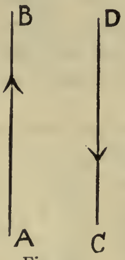


Fig. 102.

As we can prove, the field of force due to these circuits will have a vacuum between them, and therefore the circuits will be thrust towards the vacuum, and will appear to attract each other.

(*b*) If the currents flow in opposite directions, the same line of reasoning will show that the circuits will move away from each other, and will appear to repel each other.

3. **Explanation of the Mechanical Action of Magnetic Fields.**—The mechanical actions which arise between magnets and magnets, magnets and current circuits, current circuits and current circuits, are due to the stress in the magnetic ether. If we can find the direction of the lines of force in the ether due to any system of current circuits or magnets—modified it may be by the presence of soft iron—then we

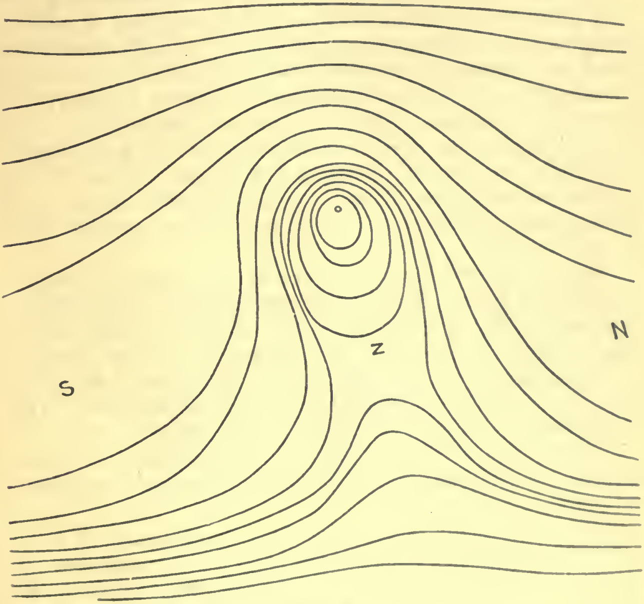


Fig. 103.

can apply the properties of these lines or tubes of induction and investigate the state of motion. It will be valuable to the student to study the subject from this point of view, and for this purpose he must carefully trace the lines of force due to different fields.

(i) Fig. 103 shows the lines of force due to a straight circuit perpendicular to the plane of the paper with the current going downwards, SN being the direction of the earth's field of force.

We see that the earth's lines of force are distorted by the lines due to the current circuit, which stirs up as it were a whirlpool in a straight stream. The lines of force due to the circuit are compressed together on one side, and open out on the other. A zero, or neutral point, or vacuum, being formed to the east of the wire. Since the tubes of induction are in a state of tension, the effect will be to move the wire perpendicularly to the plane of the paper and towards the east. The student can verify for himself the fact that this agrees with the direction of motion as determined by the method given in § 2.

Now on which side of the wire, E or W, the zero point is can easily be found. Since the current runs downwards through the paper a north pole on the east of the wire will be driven towards the south ; but this is the opposite direction in which it will be driven by the earth's magnetic action. Hence at some spot east of the wire there will be a neutral point. This found, the general nature of the lines is determined.

(ii) Fig. 104.—This figure gives the distribution of the lines of force due to a plane circuit, placed with its plane in a parallel field of force.

The general nature of the field in the neighbourhood of the wires is the same as in Fig. 103. Now we have seen that the wires move towards their neutral points, hence the circuit will rotate in a right-handed screw direction looked at from above the paper.

We see how the earth's lines have been distorted by the two circuits, and we see that the tension of these lines will cause the rotation.

Note.—If the earth's field is regarded as being from left to right, so that the right side is N, then the current is

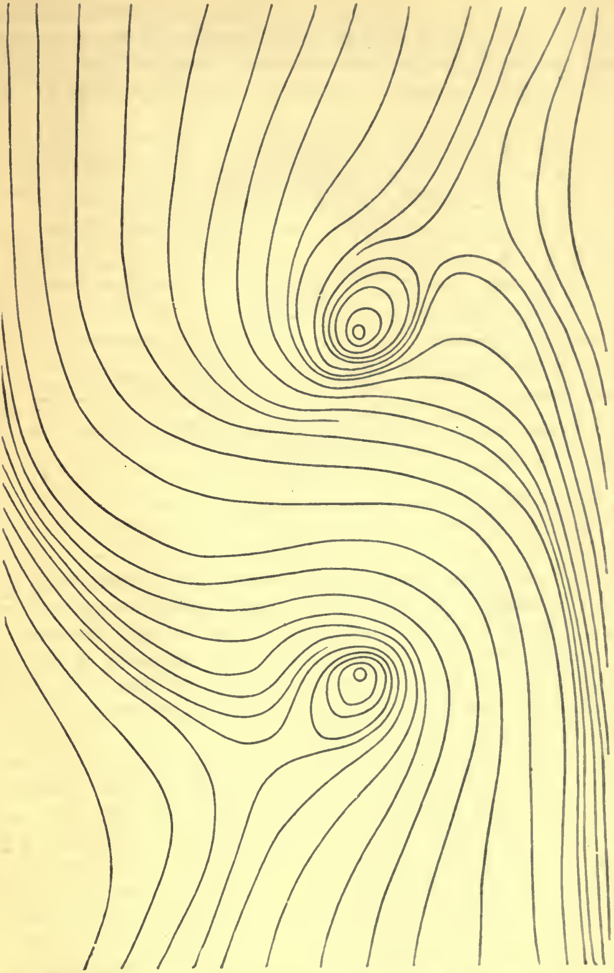


Fig. 104.

circulating through the coil from the south wire upwards through the paper and downwards at the north wire.

(iii) Fig. 105.—This figure shows the lines of force due to a circular circuit placed with its coil perpendicular to the earth's field, and with the current circulating through it in such a direction that the position is unstable. The north being to the right of the figure, if we screw along a line of force from N to S perpendicular to the coil in a right-handed direction, then this direction is the direction of the current through the coil.

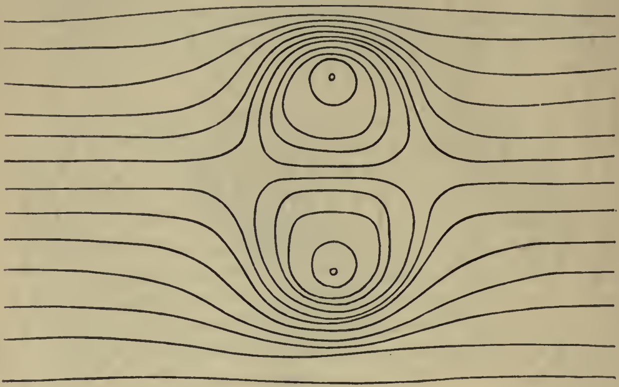


Fig. 105.

The symmetry of the figure shows the condition of equilibrium, but also suggests its instability, as there will be a tendency for the two wires to be carried round so as to give less resistance to the earth's lines.

(iv) The student may form an idea of the lines of force due to a circular current circuit placed perpendicular to a parallel field, and in the position of stable equilibrium. He will see that these lines are similar in their arrangement to a figure formed by the lines in Fig. 103, and the reflection of these lines by a mirror placed perpendicular to the paper along a N and S line to the W of the wire. In this case the neutral point will be in the plane of the coil but outside the coil. The earth's lines will be as it were squeezed through the coil, the tendency of the tension and lateral pressure of these tubes being to "burst" the coil.

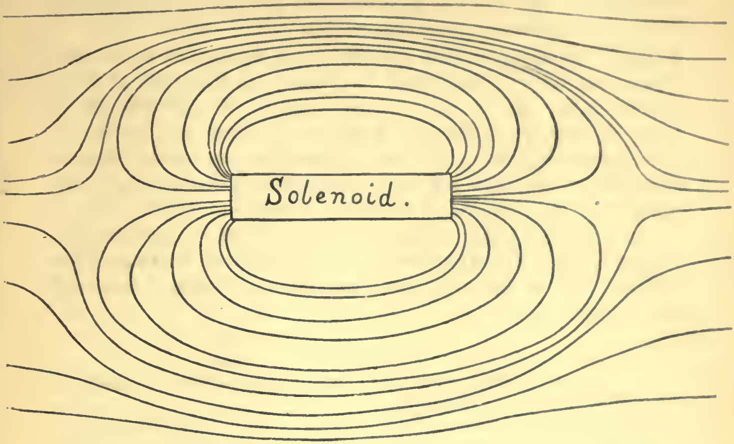


Fig. 106.

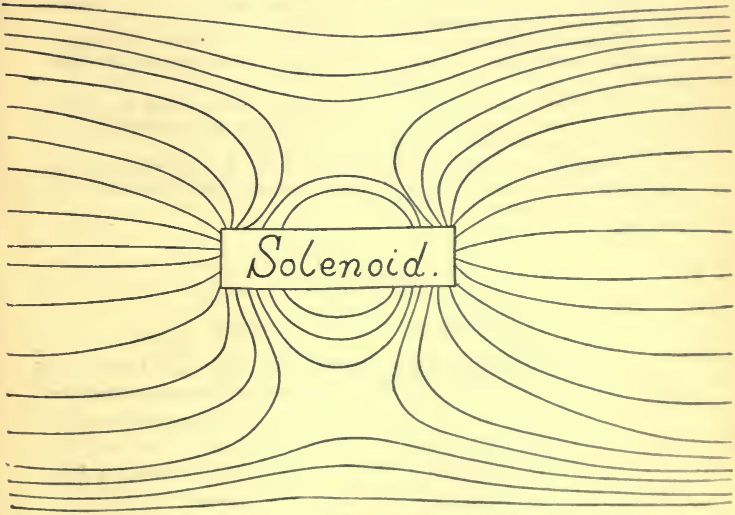


Fig. 107.

(v) Figs. 106 and 107 give the lines due to a solenoid placed with its axis in the magnetic meridian.

(vi) The action of two parallel wires upon each other may be ascertained by considering the nature of the field of force due to them. It is easy to see that if the currents are in opposite directions, the zero point or vacuum will be in the plane containing the wires but not between the wires. Hence the action of the field will be to separate the wires from each other; they will appear to repel each other.

On the other hand, if the currents are in the same direction the zero point will be between the wire, and the action of the field will be to draw the wires towards each other; they will appear to attract.

EXERCISES

1. Give Ampère's method for determining the mechanical action between two parallel circular current circuits.

2. Describe the principle of the ampere balance, and show that the force between the two coils will vary as the square of the current sent through them, if the same current is sent through each coil.

3. What will be the condition of the magnetic medium during the time the spiral in p. 210 is oscillating up and down?

4. Given the magnetic field of force due to a straight circuit, investigate directly the action between two parallel straight circuits placed near each other.

5. Sketch the lines of force when a circular current circuit is placed with its plane in a parallel field of force, and apply the properties of the lines of force to determine the motion of the circuit.

6. Sketch the lines of force due to a straight circuit placed perpendicularly in a parallel field of force, and apply the properties of the lines of force to determine the motion of the circuit.

SECTION IV

Induction of Electric Currents

1. On the Conditions necessary for an Induced Current.—Join a coil B, which consists of several turns of fairly fine wire, up to a sensitive ballistic galvanometer B.G. Now place parallel to, and coaxial with it, a second coil A, and join this coil through a suitable T.G., and through a key and a rheostat to a battery of two or three cells.

Make contact with the key so as to send a current through

A. It will be observed that the magnet of the B.G. is deflected, but that it begins to swing about its zero position. If it is a dead-beat galvanometer, and does not oscillate, the magnet will be observed to be deflected through a certain angle and then return to the zero position. This shows that a current has been created in B but has quickly disappeared. This current is said to have been induced in B by the creation of the current in A, and it can be proved that the induction only takes place so long as the current in A is changing. On breaking contact at the key a throw of the galvanometer needle in B.G. is observed to take place in the opposite direction, but equal in magnitude to that on making contact. Now the "throw" of the needle of a ballistic galvanometer is directly proportional to the quantity of electricity which has passed through the coil of the galvanometer. Hence the above experiment shows that on making and breaking the current in A, a current of electricity is induced in B, and the total quantity of electricity induced is the same in each case, but that the directions of the currents are opposite; and further that the current induced in B ceases when the inducing current in A becomes constant. The circuit A is called the **primary circuit**, and the current in it the **primary**

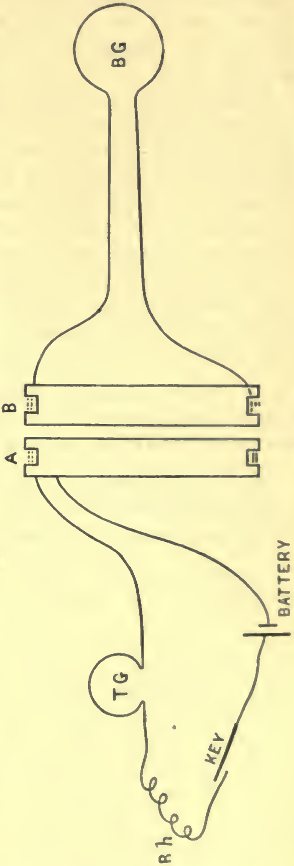


Fig. 108.

current; the circuit B is the **secondary circuit**, and the current induced in it the **secondary current**.

If we vary the current in the primary coil, the throw, and, therefore, the quantity of electricity induced in B, will vary. If the currents in A are made respectively 1, 2, 3, . . . 10 amperes, the throws produced in the B.G. will be in the ratio of 1 : 2 : 3 : . . . : 10. If, however, a piece of soft iron be introduced into the centre of the coils A, B, the quantity of electricity induced when the current in the primary is made one ampere will be much greater than when the soft iron was not there; and further, it is found that when the soft iron is present the quantity of electricity induced is not proportional to the current. The induced current does not, therefore, depend simply upon the inducing current. Faraday was the first to unravel in a long series of classic experiments the conditions under which a current is induced, and the law regulating the quantity of induced electricity. It will be convenient to state these in the form of laws. He found—

(1) **The condition necessary for induced currents is that the number of lines of induction through the secondary must be changing.**

(2) **The quantity of electricity induced is directly proportional to the change in the number of lines of induction passing through the secondary.**

The following experiments will enable the student to investigate these laws for himself:—

LAW I.—(i) Make contact with the key and observe the throw of the needle. When the needle has returned to zero, *suddenly* remove the primary to a considerable distance. It will be found that a current is induced in the secondary, and that the throw is of equal magnitude but in the opposite direction to that given on making the current. The same effect would have been produced on removing the secondary from the primary. If the primary be suddenly brought up to its original position there will be a throw of the galvanometer in the same direction, and of equal magnitude to that produced on making contact. In both cases the number of lines made to pass through the secondary is the same.

(ii) If now the primary be taken away, and a magnet be suddenly brought up to the coil, so as to increase the number of lines of force passing through the coil, a throw will be observed. On suddenly removing the magnet, there will be an equal throw in the opposite direction.

If the secondary coil B were a large solenoid, and a powerful magnet were slowly brought up to it, a current would be induced, and a fairly constant deflection produced so long as the magnet were in motion. Directly the motion of the magnet ceased, the magnetic needle would return towards zero, showing that the induction of the current only takes place whilst the number of lines of force through the secondary are changing.

(iii) Take a coil of large radius and many turns and connect it up with the ballistic galvanometer. It is convenient to have such a coil wound on a wooden circular frame, and fitted so that it can be rotated about an axis; and by moving the frame itself any desired motion can be given it. Such a coil is shown in Fig. 109, and for a reason which will appear later it is called an **Earth Inductor.** Connect this coil up to the ballistic galvanometer. Place it with its axis vertical and with its plane in the meridian. In this position no lines of force are passing through it. Now turn it quickly through 90° into a position where it is at right angles to the earth's field. A throw of the needle will be observed. Now continue to turn it in the same direction through 180°, again a throw will be observed. (The conditions which regulate the direction of the throw will be investigated later.) By varying the angles of rotation, the student will observe the varying throws. If now the axis of

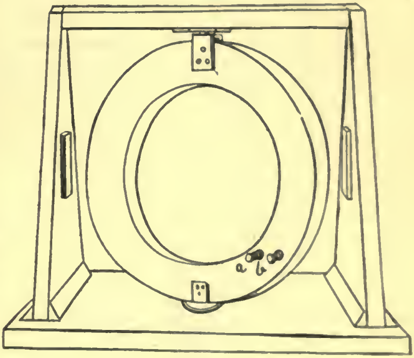


Fig. 109.

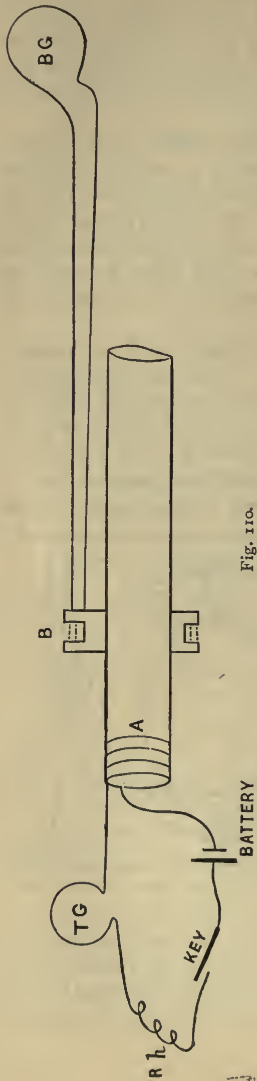


Fig. 110.

rotation be placed along the earth's line of force, *i.e.* in the magnetic meridian and along the line of dip, the lines of force in this case will run parallel to the axis and parallel to the plane of the coil whatever angle it be turned through. In fact, in this position no lines of force pass through the coil. Hence on turning the coil no throw is observed. This method may be adopted in order to find the angle of dip.

If the coil be moved parallel to itself there will be no change in the number of lines of force passing through and no throw will be observed.

These experiments show that—

(1) When there is a change in the number of magnetic lines of force passing through a coil, then there is an induced current.

(2) When there is no change, there is no induced current.

Hence Faraday's First Law is verified.

Law II.—To verify the second law it is better to take for the primary a solenoid, say 15 cm. long, and 4 or 5 cm. diameter wound with two or three layers of wire of 16 B.W.G. For secondary take a coil or bobbin which will just pass over this, and wind it with several turns of fine wire.

Now by passing currents of varying strengths through the primary, a varying number of lines of force is created in the core of the primary. Since there is no soft iron in the core or in the neighbourhood, the number of lines of force passing through the primary will be proportional to the current. Now since the secondary entirely surrounds the primary, the number of lines of force made to pass through the secondary will be equal to the number of lines of force created in the primary.

It will be found that the throw of the ballistic needle is proportional to the current in the primary, *i.e.* the quantity of electricity is proportional to the number of lines of force added to the secondary.

2. **On the Direction of the Induced Current.**—We have seen that when a magnet, say a north pole, is brought up to a coil, a current is induced in the coil. Now the energy required must be derived from the work done in bringing up the magnet, and as the magnet is brought up to the coil it must experience a resistance. Hence the magnetic action of the induced current must be such as to oppose the motion. This gives a method of determining in any particular case the direction of the induced current. It was given in this form by Lenz, and is usually known as Lenz's Law of Induction.

LENZ'S LAW.—**The direction of the induced current is always such that its magnetic action opposes the change producing the current.**

If there are no lines of force through the coil before the magnet is brought up, then since the lines of force due to the induced current must be in the opposite direction to the lines of force introduced by bringing up the magnet, we see that the tendency is to keep the number of lines of force constant. In fact the coil offers resistance to having the number of lines of force through it changed—much the same as a body offers resistance to having its velocity changed—or an elastic body to having its form or volume changed. When, however, work is done on the medium in putting it in the given state of stress with reference to the coil, *i.e.* when work is done in creating these lines of force through the coil which are said to be due to the induced current, the coil possesses energy and is capable of doing work in return.

Applications of Lenz's Law.—(1) If the current in the primary in Expt. i. p. 224 is right-handed, looked at from the left, then the equivalent shell will be S on the left and north on the right. Hence the left face of B facing the primary A must be N to oppose the north of the adjacent face of A, hence the current through B must be left-handed.

(2) If the north pole of a magnet be brought up to a coil, the induced current created must be such as to give rise to a magnetic shell opposing the approach of the north pole—*i.e.* must have a north face towards the north pole of the magnet. Hence the direction of the induced current must be left-handed looked at from the north pole of the magnet.

(3) *To consider the direction of the current in an earth inductor which is made to rotate in the earth's field of force about a vertical axis; and further, to consider the direction of the magnetic force at the centre of the coil due to the induced current.*

(i) Let SN be the direction of the earth's meridian. Let

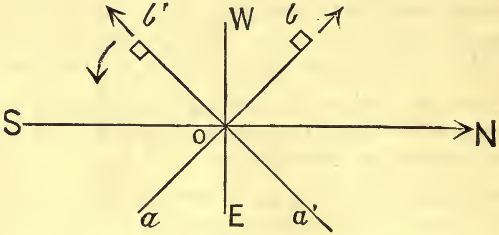


Fig. 111.

ab be the coil the front of which is marked with the square. As the coil rotates about an axis perpendicular to the paper in a right-handed direction (*i.e.* contrary to the hands of a watch placed with its face upwards) the square moving from N towards the S through W, work must be done. Hence the equivalent shell due to the induced current must have its face N and back S, hence the induced current must be left-handed. Hence in the first quadrant as *b* moves from N through a quadrant, the induced current is left-handed (looked at from the front). Hence the intensity at the centre due to this induced current is perpendicular to the coil in the position it is in at the moment, and is in the direction Ob' (Fig. 111). Hence as the coil rotates through the first quadrant, the intensity at the centre acts from O in a SW direction.

(ii) As the coil moves through the second quadrant, the face at b' moving towards the south, the equivalent magnetic shell must have its face S, since the shell being magnetised perpendicularly to itself, the south pole of each small magnet of the shell is being turned away from the south, and hence work must be done. Hence looked at from the face the direction of the current in the second quadrant is right-handed, *i.e.* the direction of the current has changed. Now the intensity at the centre, since the current is right-handed, will be from face to back, *i.e.* in the direction Ob, *i.e.* as the coil is rotating in the second quadrant, the intensity at the centre is in a NW direction.

(iii) As the coil moves through the *third* quadrant, the equivalent shell must have its face S, as this face is still being turned towards the north. Hence the current is right-handed as in the second quadrant. Again, since the intensity at the centre of the coil due to the induced current is perpendicular to the coil and is from face to back, the direction of the intensity is along Oa' (Fig. 112), *i.e.* in a SW direction.

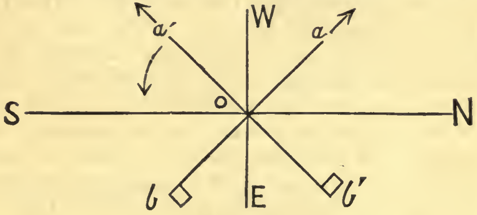


Fig. 112.

(iv) As the coil moves through the fourth quadrant the equivalent shell must have its face N, as this face is now being turned away from the north. Hence looked at from the face the current is left-handed. Also the intensity at the centre due to the induced current is from back to face, *i.e.* along Oa (Fig. 112), *i.e.* in a NW direction.

The result of this investigation is—

(a) As regards the current it is right-handed in the second and third quadrants, and left-handed in the first and fourth quadrants, being reversed each time the plane of the coil passing at right angles to the meridian.

(*b*) As regards the intensity at the centre due to the induced current which is created as the coil rotates in a left-handed direction, the direction is always towards the west. Hence if the coil is rotated so fast that the series of impulses given to the north pole placed at the centre form a steady force, the direction of this force is towards the west.

If therefore the coil be rotated in the earth's field a magnet pivoted at the centre will be turned from the meridian in the direction of rotation of the coil. If the coil is rotated at a uniform rate, sufficiently fast, the magnet will take up a permanent deflection.

3. **On the E.M.F. of the Induced Current.**—It can be shown by dynamical reasoning that the E.M.F. in the secondary circuit at any moment is measured by the rate of change of the number of lines of force passing through the circuit. Hence by making the number of lines of force change at as great a rate as possible, the induced E.M.F. can be made very great. Use is made of this fact in two important electrical instruments. The "Induction Coil" is used to transform a low E.M.F. into a very high E.M.F., and the "transformer" is used, generally, to transform a *high* alternating E.M.F. into a *low* alternating E.M.F.

4. **Self-Induction.**—Since a current is induced in a circuit when the number of lines of force through the circuit is changing, it is natural to expect that as the current in a circuit changes (*e.g.* at make of the circuit as the current increases to a maximum), the number of lines of force which passes through the circuit due to the current in the circuit itself will change, and hence there will be an induced current on account of this change. This phenomenon is called **self-induction.** When the current is increasing, the lines of force are increasing, and the magnetic action of the induced current will be such as to oppose this increase, hence there will be a current induced in the circuit in the opposite direction to the increasing current. When the current is decreasing, as at break, the lines of force

will be decreasing, and the magnetic action of the induced current, will be such as to create more lines of force in the same direction as that due to the current, and hence the induced current will be in the same direction as the diminishing current. In the first instance the effect of the induced current will be to retard the increase, in the second instance at break the effect of the induced current will be to accelerate the decrease of the current. The induced E.M.F. at break will be much greater than the E.M.F. at make. Hence the current will be able to spark across a greater space at break than at make. This accounts for the spark being greater at break than at make, which is especially noticeable if there is a strong electromagnet in circuit, for in this case a strong magnetic field is created.

The phenomenon may be looked at from another point of view. During the time the current is increasing, the electromagnet is being created, and a magnetic field charged as it were with lines of force, *i.e.* put into a state of stress, and hence energy created. When the discharge takes place at break the energy of the electromagnet supplies an additional current, and consequently greater E.M.F.

Practical applications of induced currents

1. **The Telephone.**—A thin sheet M of ferrotype iron is fixed in front of a cylindrical magnet. A bobbin B wound with fine wire is placed on the end of the magnet. An exactly similar apparatus is used at the other end of a line of wire connecting the two bobbins of coils. Now by

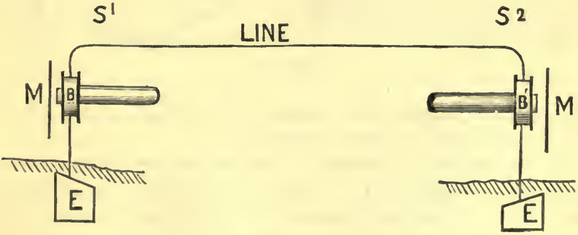


Fig. 113.

speaking into the trumpet-shaped piece the vibrations set up in the air set up corresponding vibrations in the ferrotype iron. The motion of the iron in front of the pole of the magnet changes the number of lines of force passing through the coil. Hence induced currents are set up in the coil, and these induced currents passing through the similar coil or the second instrument cause the magnetic field through the coil to vary, and hence the ferrotype iron disc is set in motion, and vibrations similar to the original vibrations are set up in the air ; but with reduced intensity on account of the loss of energy by the way.

2. **The Microphone.**—This instrument depends upon the fact that any change in the resistance of the primary circuit

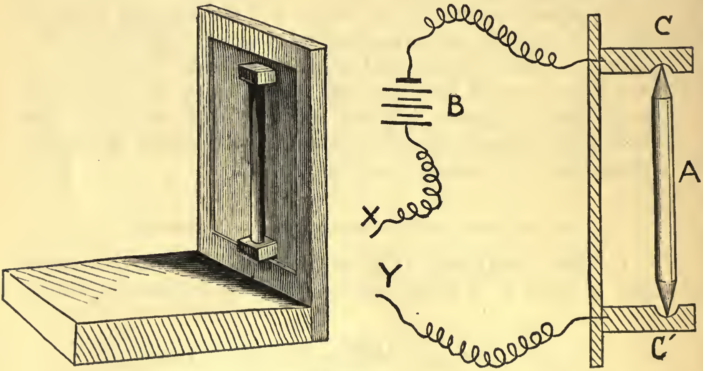


Fig. 114.

will cause a change of current in the primary, and will set up an induced current in the secondary.

It consists of a cylinder of carbon resting loosely between two metallic supports CC′. The primary current passes through the carbon, *i.e.* the microphone is placed in the primary circuit. Now any shaking of the carbon will cause slight change in the resistance at the point of contact, and hence induced currents in the secondary.

3. **The Dynamo.**—We have seen that if a coil be rotated in a field of force induced currents are generated, which are

reversed every half revolution. This is essentially a dynamo supplying alternating currents.

As has been explained under the electromotor by means of a commutator these alternating currents can be sent in the same direction through the external circuit, and the dynamo converted into a direct current machine.

4. **The Ruhmkorff's Induction Coil.**—This consists of an electromagnet formed by winding thick copper wire (the primary coil) round a cylindrical soft iron core. Then a coil, forming the secondary, of very fine insulated wire is wound over the primary. If now a current be alternately made and broken in the primary at a great rate, a large E.M.F. will be induced in the long coil of the secondary. Hence if the two ends of the secondary are brought to two metallic points, a spark can be made to pass between the point through the intervening air space. The make and break can be worked by an automatic soft iron hammer of the same kind as shown in the electric bell.

5. **The Transformer.**—This consists in principle of an induction coil such as Ruhmkorff's, but in many forms of the machine the soft iron core is a closed circuit. An alternating current of high E.M.F. is sent through the long fine wire coil, and this gives rise to an alternating current in the short thick coil of lower E.M.F. but correspondingly larger current.

EXERCISES

1. What are the conditions necessary for an induced current? Describe experiments.

2. Upon what does the quantity of the electricity induced depend? Give experiments.

3. A north pole is brought up to a closed coil, show how to find the direction of the induced current.

4. Show how to compare the strengths of two magnetic poles by means of induced currents.

5. Give Lenz's law and show how it follows from the principle of the conservation of energy.

6. Explain why a magnet placed at the centre of a coil which is rotated about a vertical axis is turned from the meridian in the direction of the rotation of the coil.

7. Show how to verify by experiment that the quantity of electricity induced is proportional to the increase of the number of lines of magnetic force through the coil.

8. A coil in which a current is passing is moved up to a coil and parallel to it, show how to find the direction of the induced current.

9. Explain the formation of self-induced currents. Why is the spark at the break greater than at the make? and why does the introduction of a powerful electromagnet in the circuit considerably increase the spark?

10. Upon what does the E.M.F. of the induced current depend? What is the principle of the "Ruhmkorff's coil," and of the "Transformer?"

Exercises on Chapter IV

1. A rod of soft iron is placed upright on a table. Its upper end is surrounded by a coil of insulated wire which does not touch the rod. When a strong current goes through the wire, the iron rises in the coil. Explain how.

2. Explain the effect of introducing an iron core into a wire helix conveying a current.

3. A ruler balanced at its centre has a current passed along a spiral of wire wound symmetrically round it from end to end. Show how you would ascertain, by a bar magnet, the direction of the current in the wire.

4. How may an electric current be employed to magnetise a bar of iron or steel? What difference of behaviour will be shown by iron and steel when the current ceases to flow?

5. Wires from two separate voltaic batteries are stretched one above the other from north to south (magnetic), and equal currents pass through both wires. If a magnetic needle, free to turn horizontally but not vertically, is hung halfway between the wires, how will it be affected—

(*a*) if the currents are both in the same direction?
(*b*) if the currents are in opposite directions?

6. A long copper wire covered with silk is wound several times round an iron rod. On connecting the ends of the wire with the terminals of a Daniell's battery, the iron rod becomes a magnet. How does the direction of magnetisation of the iron (or position of its north-seeking and south-seeking poles) depend upon how the copper wire is wound, and which end of it is connected with the copper end of the battery? Give a drawing.

7. A gutta-percha covered copper wire is wound round a wooden cylinder AB from A to B. How would you wind it back from B to A (1) so as to increase, (2) so as to diminish the magnetic effects which it produces when a current is passed through it? Illustrate your answer by a diagram drawn on the assumption that you are looking at the end B.

8. A circuit consisting of a battery and an electromagnet can be made and broken by a wire dipping into mercury. At every break a bright spark occurs. If the electromagnet be removed from the circuit, the spark is much less bright. Explain these phenomena.

9. What are Ampère's rules respecting a wire conducting a current and a magnetic needle acted upon by the current flowing in the wire?

10. What is meant by Electromagnetic Induction; and what is Lenz's law? Give examples.

11. Write down Lenz's law of induction, and describe an experiment that exhibits the inductive action of a magnet on a non-magnetic body

when the one moves in presence of the other, showing that this action conforms to Lenz's law.

12. Investigate the magnetic field in the neighbourhood of a straight wire conveying an electric current, and explain the behaviour of a magnetised needle freely suspended near it.

13. Show how a magnet may be made to rotate continuously by the action of an electric current, and point out the direction of rotation in any one case.

14. Explain the damping action of a copper plate on a vibrating magnetic needle.

15. Show that by means of a flat coil of wire, provided with an arrangement for rotating it in any plane, and for connecting it with a galvanometer, it is possible to determine the magnetic inclination at any place.

16. When a circular metallic hoop is rotated in the earth's magnetic field, electric currents are generally produced in it. In what positions of the axis of rotation will the induced currents be the greatest and least respectively? Give reasons for your answer.

17. It is sometimes said that an electric current behaves as though it possessed *inertia*. Discuss this statement.

18. A hoop of wire is made to rotate uniformly about a vertical axis. Show the direction and variation in strength of the current generated in it at different periods of its revolution.

19. Describe an induction coil. What relation exists between the direction of the inducing current and the induced electricity?

20. Explain the principle of the ordinary dynamo-electric machine. What is meant by (1) shunt-wound, (2) series-wound, (3) separately excited?

21. Explain the principles involved in the simplest telegraphic signalling.

22. Describe a telephone and explain its action.

23. Describe the construction and action of an electric bell.

24. Describe the construction and action of any form of dynamo of the "magneto" type [*i.e.* in which the field magnets are permanent steel magnets].

25. A coil of wire moves in a magnetic field. Under what conditions will there be, and under what conditions will there not be, a resultant E.M.F. induced in the coil?

26. In what way may an electric current be induced, and on what does its strength depend?

27. Describe briefly the Gramme Machine, and show how electrical energy is generated by the expenditure of mechanical work.

APPENDIX

A. The following recapitulation of dynamical results will be found convenient :—

1. One **foot** =30.48 centimetres.
One **pound**=453.59 grammes.

2. 1 foot-per-sec. =30.48 cm.-per-sec.
1 ft.-per-sec.-per-sec. =30.48 cm.-per-sec.-per-sec.

The acceleration of a falling body may be taken as **32.2 ft.-per-sec.-per-sec.**, *i.e.* **981 cm.-per-sec.-per-sec.**

3. A **poundal** is the force required to create in one *pound* an acceleration of one *ft.-per-sec.-per-sec.*

A **dyne** is the force required to create in one *gramme* an acceleration of one *cm.-per-sec.-per-sec.*

Now the force required to create in 453.59 gm. an acceleration of 30.48 cm.-per-sec.-per-sec. is 453.59 × 30.48 dynes, *i.e.* 13825.4 dynes.

But this force is the poundal,

Therefore 1 **poundal**=13825.4 **dynes.**
1 pound-weight (lb.-wt.) =32.2 poundals.
1 gramme-weight (gm.-wt.)=981 dynes.
Now 1 lb.-wt. =453.59 gm.-wt.
=453.59 × 981 dynes.
=444961 dynes.

4. A **foot-poundal** is the work done in overcoming a resistance of one poundal through a distance of one foot.

A cm.-dyne or **erg** is the work done in overcoming a resistance of one dyne through a distance of one centimetre.

To overcome 13825.4 dynes (*i.e.* one poundal) through a distance of 30.48 cm. (*i.e.* one foot) requires 13825.4 × 30.48 ergs, *i.e.* 421400 ergs.

But this work is a foot-poundal,

Therefore One **ft.-poundal**=421400 **ergs.**

One ft.-lb.-wt. is the work required to overcome one lb.-wt. through one foot, *i.e.* 453.59 gm.-wt. through 30.48 cm., *i.e.* 453.59×981 dynes through 30.48 cm., and is therefore equal to $453.59 \times 981 \times 30.48$ ergs, *i.e.* 13562700 ergs or 1.356×10^7 ergs.

1 **joule** $= 10^7$ ergs, *i.e.* 10 million ergs.

Therefore 1 ft.-lb.-wt. = 1.356 joules.

$$\text{One joule} = \frac{1}{1.356}\ \text{ft.-lb.-wt.} = .737\ \text{ft.-lb.-wt.}$$

5. A **Horse-power** is equal to 33000 ft.-lbs.-wt. per min., *i.e.* 550 ft.-lbs.-wt. per sec.

Therefore

$$\begin{aligned} 1 \text{ Horse power (H.P.)} &= 550 \text{ ft.-lbs.-wt.-per-sec.} \\ &= 550 \times 1.356 \text{ joules-per-sec.} \\ &= 745.8 \text{ joules-per-sec.} \end{aligned}$$

A joule-per-sec. is called a **Watt.**

∴ 1 H.P. = 746 watts (nearly).

6. The quantity of heat required to raise the temperature of one pound of water from 4° C. to 5° C. is called the water pound Centigrade degree unit of heat, or briefly the (w.lb.C.°) unit.

Similarly there are the pound Fahrenheit, and the gramme Centigrade units. The relations between them are

$$\begin{aligned} 1 \text{ w.lb.C.}^\circ &= 453.59 \text{ w.gm.C.}^\circ \\ 1 \text{ w.lb.F.}^\circ &= \frac{5}{9} \text{ w.lb.C.}^\circ \\ &= \frac{5}{9} \times 453.59 \text{ w.gm.C.}^\circ \\ &= 251.98 \text{ w.gm.C.}^\circ \end{aligned}$$

Dr. Joule in 1867 and again in 1878 found by experiment that to raise the temperature of one pound of water through one Fahrenheit degree required an expenditure of 772 ft.-lbs.-wt. of work. More recent experiments (Rowland in 1879, and Griffiths in 1893) have shown that a nearer value is 778 ft.-lbs.-wt. per w.lb.F.° This quantity is called the dynamical equivalent of heat.

Now 1 Fahrenheit degree $= \frac{5}{9}$ Centigrade degree, therefore it will require $778 \times \frac{9}{5}$ (*i.e.* 1400) ft.-lbs.-wt. of work to raise the temperature of 1 lb. of water through 1 Centigrade degree (*i.e.* through $\frac{9}{5}$ F.°).

Hence Joule's equivalent is 1400 ft.-lbs.-wt. per w.lb.C.°

Again, 1 ft.-lb.-wt. = 1.356 joule.

Therefore 1400 ft.-lbs.-wt. = 1400×1.356 joules, Hence to create 1 w.lb.C.° requires 1400 × 1.356 joules, and therefore to create 1 w.gm.C.° requires $\frac{1400 \times 1.356}{453.59}$ joules, *i.e.* 4.189 joules.

Hence Joule's dynamical equivalent is

4.189 joules per w.gm.C.°

Or approximately 4.2 joules per w.gm.C.°

Hence 1 w.gm.C.° = 4.2 joules.

1 joule = .24 w.gm.C.°

B. REFLECTING AND BALLISTIC GALVANOMETERS.

For some of the experiments in Chapter IV. § 4 a more sensitive form of galvanometer than the galvanometer described in Chapter II. is required. A simple form of such a galvanometer is shown in Fig. 115.

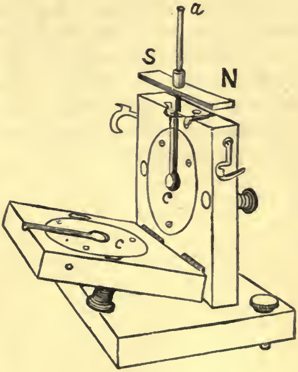


Fig. 115.

Two coils are wound on a metal frame, and are then well soaked in melted paraffin wax. When the paraffin wax cools it binds the turns of the coils firmly to each other so that they can be taken out of their frames and fixed to two pieces of wood.

These wooden holders are fixed vertically to a square base board, which is provided with three levelling screws. The two coils are close together and coaxial. The magnet is fixed to the back of a small concave mirror which is suspended at the centre of the two coils by a single cocoon fibre. It is convenient that the wooden frame containing one coil should be hinged to the other, as is shown in the figure, in order that the suspended magnet may be easily reached. Above the coils is placed a control magnet, by means of which the magnet and direction of the field at the centre of the coils can be varied.

It can be proved mathematically, and can be verified by experiment, that the sine of half the angle of throw is directly proportional to the total quantity of electricity which has passed through the coil. If the angle of throw is very small, the sine of half the angle of throw is proportional to the deflection as shown on the screen. Hence for such a galvanometer under the condition that the current passes in a time which is very small compared with the time of vibration of the magnet, the quantity of electricity is proportional to the deflection of the throw, as shown on the screen. A galvanometer which is used in this way to measure quantity of electricity is called a Ballistic Galvanometer.

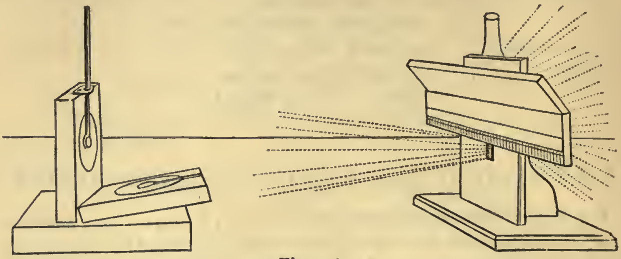


Fig. 116.

In performing the experiment in § 4 the changes must be made rapidly so that the whole induced current passes before the needle begins to move (appreciably) from its zero position.

Qualitative results can, however, be obtained without a massive needle galvanometer.

TABLE OF TANGENTS

	Minutes.					Minutes.			
Deg.	0′	15′	30′	45′	Deg.	0′	15′	30′	45′
1	.0174	.0218	.0262	.0305	46	1.0355	1.0446	1.0538	1.0630
2	.0349	.0393	.0437	.0480	47	1.0724	1.0818	1.0913	1.1009
3	.0524	.0568	.0611	.0655	48	1.1106	1.1204	1.1303	1.1403
4	.0699	.0743	.0787	.0831	49	1.1504	1.1605	1.1708	1.1812
5	.0875	.0919	.0963	.1007	50	1.1917	1.2024	1.2131	1.2239
6	.1051	.1095	.1139	.1183	51	1.2349	1.2460	1.2572	1.2685
7	.1228	.1272	.1316	.1361	52	1.2799	1.2915	1.3032	1.3150
8	.1405	.1450	.1494	.1539	53	1.3270	1.3391	1.3514	1.3638
9	.1584	.1629	.1673	.1718	54	1.3764	1.3891	1.4019	1.4150
10	.1763	.1808	.1853	.1898	55	1.4281	1.4415	1.4550	1.4687
11	.1944	.1989	.2034	.2080	56	1.4826	1.4966	1.5108	1.5252
12	.2125	.2171	.2217	.2263	57	1.5399	1.5547	1.5697	1.5849
13	.2309	.2354	.2401	.2447	58	1.6003	1.6160	1.6318	1.6479
14	.2493	.2540	.2586	.2632	59	1.6643	1.6808	1.6977	1.7147
15	.2679	.2726	.2773	.2820	60	1.7320	1.7496	1.7675	1.7856
16	.2867	.2915	.2962	.3010	61	1.8040	1.8227	1.8418	1.8611
17	.3057	.3105	.3153	.3201	62	1.8807	1.9007	1.9210	1.9416
18	.3249	.3297	.3346	.3394	63	1.9626	1.9840	2.0057	2.0278
19	.3443	.3492	.3541	.3590	64	2.0503	2.0732	2.0965	2.1203
20	.3640	.3689	.3739	.3789	65	2.1445	2.1692	2.1943	2.2199
21	.3839	.3889	.3939	.3989	66	2.2460	2.2727	2.2998	2.3276
22	.4040	.4091	.4142	.4193	67	2.3558	2.3847	2.4142	2.4443
23	.4245	.4296	.4348	.4400	68	2.4751	2.5065	2.5386	2.5715
24	.4452	.4505	.4557	.4610	69	2.6050	2.6394	2.6746	2.7106
25	.4663	.4716	.4770	.4823	70	2.7475	2.7852	2.8239	2.8636
26	.4897	.4931	.4986	.5040	71	2.9042	2.9459	2.9887	3.0326
27	.5095	.5150	.5206	.5261	72	3.0777	3.1240	3.1716	3.2205
28	.5317	.5373	.5429	.5486	73	3.2708	3.3226	3.3759	3.4308
29	.5543	.5600	.5658	.5715	74	3.4874	3.5457	3.6059	3.6679
30	.5773	.5832	.5890	.5949	75	3.7320	3.7982	3.8667	3.9375
31	.6009	.6068	.6128	.6188	76	4.0108	4.0867	4.1653	4.2468
32	.6249	.6309	.6371	.6432	77	4.3315	4.4194	4.5107	4.6057
33	.6494	.6556	.6619	.6682	78	4.7046	4.8077	4.9151	5.0273
34	.6745	.6809	.6873	.6937	79	5.1445	5.2671	5.3955	5.5301
35	.7002	.7067	.7133	.7199	80	5.6713	5.8196	5.9758	6.1402
36	.7265	.7332	.7399	.7467	81	6.3137	6.4971	6.6911	6.8969
37	.7535	.7604	.7673	.7743	82	7.1154	7.3479	7.5957	7.8606
38	.7813	.7883	.7954	.8026	83	8.1443	8.4489	8.7769	9.1309
39	.8098	.8170	.8243	.8317	84	9.5144	9.9310	10.3854	10.8829
40	.8391	.8466	.8541	.8616	85	11.4300	12.0346	12.7062	13.4566
41	.8693	.8770	.8847	.8925	86	14.3007	15.2570	16.3498	17.6105
42	.9004	.9083	.9163	.9244	87	19.0811	20.8188	22.9037	25.4517
43	.9325	.9407	.9490	.9573	88	28.6362	32.7303	38.1885	45.8294
44	.9657	.9741	.9827	.9913	89	57.2899	76.3900	114.5886	229.1816
45	1.0000	1.0087	1.0176	1.0265	90	∞			

LOGARITHMS OF NUMBERS FREQUENTLY OCCURRING

log. 30.48 = 1.4840150
log. $(30.48)^2$ = 2.9680300
log. $(30.48)^3$ = 4.4520450
log. 453.59 = 2.6566635
log. π = .4971509
log. π^2 = .9943018
log. 981 = 2.9916690
log. 32.2 = 1.5078559
log. 4.2 = .6232493

ANSWERS TO EXERCISES

Page 42

1. $208.\dot{3}$; 600, 1127.6, 1181.7. **2.** 13048. **3.** 4×10^6, 2×10^6, $.6944 \times 10^6$ dynes. **5.** .0317, .0655, .1039, .1510; .1827, .1915, .2078, .2349.

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4. 25 : 36. **5.** 13 : 28. **6.** 675. **7.** 13 : 28. **8.** .2176, .0648, .4608, .405. **9.** 25 : 36. **10.** 2.08×10^6 (see p. 57). **11.** $\frac{41}{16}$ H, $\frac{7}{16}$ H.

Page 54

1. 243.01. **2.** $\frac{2}{5}$ dyne. **3.** .00107 dyne. **4.** 7, 5.47. **5.** $4.\dot{4}$, 1.97, 2.47. **6.** 3837500, 17.865, .72. **7.** 243.01. **8.** 939.6. **9.** 8.16. **10.** $\frac{\sqrt{2}}{2}$, $\frac{\sqrt{3}}{3}$ respectively of first distance apart. **11.** 60. **12.** 1305.5. **13.** 40. **15.** 7.71. **16.** .0333. **17.** 2.23 : 1. **18.** 18° nearly, 27° nearly. **19.** .0685, .1203, .1888. **21.** 8201.25. **22.** 1 : 4. **24.** 7.07, 70.7. **25.** $1666.\dot{6}$. **26.** 9.8. **27.** 4.

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9. 7°.37′. **18.** 2 : 3. **21.** Twice round. **24.** .2592, .1152.

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5. .4, .8, .1, .2, .4. **8.** 5. **9.** .252 amperes, 14° 48′, 49° 37′. **10.** 1.35 amperes. **11.** 31415.9. **14.** 46° 59′. **15.** .1997, .00748, .000149, .000000497.

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5. 8.8 gm. of sulphate, 3.5 gm. of Cu; $\frac{1}{9}$ gm. of hydrogen, $\frac{8}{9}$ of oxygen. **6.** 7 : 8. **8.** .002884, .003347, .003656, .001648, .000309, .005047, .003759, .001854, .006489, .006386. **9.** 3.5748. **10.** 9.826. **11.** 13.4 amperes. **12.** .000333. **13.** .6255 amperes, .367 gm. **14.** .678. **15.** 4.8 amperes. **16.** 2.026 amperes. **17.** 7.257, 17.64.

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4. 4.2×10^6, 2.675×10^7, 2.857×10^7. **6.** 12600 joules, 2400 coulombs, 5.25 volts. **10.** 2976 joules. **11.** 4.798 d. **12.** 180.47 gm., 1.59 d. **14.** 64 watts, .0858 H.P., .45 lbs.

PAGE 122

5. 72 volts. **6.** 40 ohms. **7.** 156.2 ohms. **8.** 1.4 volts, .7 ohms. **9.** 2.8 volts, 24 C.° **10.** 3.21 C.° **11.** 20.48 joules-per-sec., or 4.87 w.gm.C.° per sec. **12.** 50 joules-per-sec. **13.** .1 ohm. **14.** 3 : 4.

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1. 1, $\frac{6}{11}$, 1.986. **2.** .00645. .0000665. **3.** .243 ohm nearly. **4.** 3 : 1 nearly. **5.** .412 ohm, 9.728 volts. **6.** 1.646 ohms. **7.** 41°. **9.** 1.2 : 1.

PAGE 140

1. $\frac{30}{31}$, $\frac{3}{31}$, $7\frac{8}{11}$. **2.** 11, 101, 51. **3.** $\frac{6}{11}$; $2\frac{1}{5}$, $2\frac{3}{4}$, $3\frac{2}{3}$, 6. **4.** $\frac{341}{181}$. **5.** $\frac{75}{31}$, $\frac{50}{31}$, $\frac{30}{31}$. **6.** $\frac{3}{610}$, $\frac{18}{61}$. **7.** $\frac{10}{27}$, $\frac{4}{27}$, $\frac{6}{27}$. **8.** 1.6. **9.** 17.64. **10.** $\frac{E}{15}, \frac{7E}{55}, \frac{2E}{55}$.

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1. .269, 269.6, .8166. **2.** 3906.7 cm., 725.81. **3.** 1677.8×10^{-9} ohms-per-cm.-sq.-cm. **4.** 10.617 cm. **5.** 40960. **6.** 84.726 ohms. **7.** 21535, 6542.5, 804.73, 149.56 cm. **8.** .00464, .01528, .3904, 2.1006.

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1. 2.06 : 1. **2.** 1.226 : 1. **3.** 2 : 1. **4.** 2 : 1. **5.** 2.53 : 1. **6.** (1) All in series; (2) either all in series or 50 in series 2 in arc; (3) 50 in series 2 in arc; (4) 50 in series 2 in arc; (5) 25 in series 4 in arc *or* 20 in series 5 in arc; (6) 20 in series 5 in arc *or* 10 in series 10 in arc; (7) 4 in series 25 in arc; (8) all in arc; (9) all in arc. **7.** 4 in series 5 in arc.

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5. $\frac{1}{6}$, $1\frac{5}{6}$, $\frac{3}{11}$. **6.** $\frac{1}{30\pi}$ microfarad. **7.** $\frac{K}{450}, \frac{K}{360}$ microfarads. **8.** $\frac{1}{45000}$, $\frac{1}{18000000}$, $\frac{1}{9000000}$ microfarads. **9.** $\frac{3 \times 10^{26}}{\pi}$ dynes $\frac{\pi}{3} \times 10^4$ dynes.

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